## NORBERT GIESLER

# New earth pressure teachings with updated calculations 

Concise version


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## Concise version

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## Preface

All over the world, we can marvel at numerous structures that document the architectural prowess of their respective dates of origin. But they also demonstrate how further development of the existing state of the art is jointly promoted by the expertise, abilities, and experience of scientists, engineers, architects, and craftsmen. In this way, planning documents and illustrative examples were created for the next generations.

Also Monsieur de Coulomb's (1736-1806) classical earth pressure theory has been repeatedly modified and adapted to newer findings.

But in our digital and global world, planning documents must be exactly reproducible, and empirical knowledge must be included in the basic planning of experts.

The new earth pressure teachings are intended as a contribution to this development, and in particular to help prevent future damage events in structural and civil engineering.

This aim is supported by the "Studie Erddruck" ("Study on earth pressure"), the book "Die neue Erddrucklehre auf den Grundlagen der reinen Physik", ISBN 03831-122-978-3-5 ("New earth pressure theory based on pure physics"), and various technical papers on the subject. This concise version of the book has been supplemented with numerous updated calculation examples. Study and book can be downloaded free of charge from www.erddruck.de.

Wide discussions about the new findings in earth pressure teachings are an important step towards obtaining safe planning documentation.

The author
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## List of symbols

Terms and symbols are detailed in my 'Earth pressure study' and in the book. In this work, I have omitted the list - instead, the symbols are explained directly on the object.

## 1 Introduction

### 1.1 General information on the present state of affairs

Repeatedly, we hear reports in the media about damage to structures subjected to soil pressures, such as supporting walls, underground pipes, and tunnel ceilings. Similarly, reports about landslides are increasing dramatically. Apart from the high material damage costs of these events, quite often there are casualties with severe of fatal injuries. Particularly noteworthy in 2009 were the collapse of the Historic Archive in Cologne and the gigantic rockslide in Nachterstedt. More recently, we heard about subsidence of the Autobahn A 20 near Triebsees, and the bumpy road in Husum. On the Autobahn in Geiselwind, the new noise protection embankments must be demolished and rebuilt over a distance of 1.3 km . After such damage incidents, the question of who is to blame arises very quickly, and there is talk about botched up construction.

However, the 'culprits' usually had simply worked according to the relevant German rules and regulations, which are demonstrably faulty. Although the application of standards, rules, and regulations is not mandatory, they are expressly demanded from planners and construction companies in the specifications and/or construction contracts. Moreover, the current earth pressure teachings and Eurocode 7 are viewed as generally accepted engineering practice. Consequently, they must be observed. As a result, their discrepancies contribute decisively towards under-dimensioning of earth embankments and structures subjected to soil pressures.

### 1.2 Purpose and structure of this work

Findings about soil behaviour in free nature and die predictability of soil properties resulted in an investigation of the discrepancies in current earth pressure teachings. Contradictions between natural soil behaviour and the calculation requirements were found, and by applying basic physics they were proved to be wrong. However, the accumulation of these discrepancies makes it impossible to simply correct the writings of the teachings. In order to make comparisons between the current and new earth pressure teachings, new terms and symbols have been introduced.

### 2.0 Earth pressure, innovations in the calculation defaults

3.0 Earth pressure, more detailed calculation examples
4.0 Summary

Due to these changes in the calculation defaults, it would be possible to minimise damage in civil engineering projects.

### 1.3 Material and methods

To illustrate the current earth pressure teachings, writings from the Center for Geotechnics at the Technical University Munich (TUM) were used.

Own experience and know-how led to the development of a new procedure for the determination of soil properties. Accordingly, two different force systems occur in soils. They depend on the center of gravity in the earth wedge, and lead to different calculation results.

## 2. Earth pressure - innovations in calculation defaults

### 2.1 Different force systems in soils

The realization that two different force systems exist for determining earth pressure is highly significant for earth pressure teachings. This is shown in the operation of an hourglass.
If sand is filled into the lower container of the hourglass, a bulk cone is formed on the horizontal base. The sand in the cone behaves inactively and only changes its shape under the influence of an external force. Its center of gravity is located in the lower third of its height $h$. If the hourglass is rotated through $180^{\circ}$, the sand (restrained by the container wall) adopts the shape of a 'cone standing on its point'. Its center of gravity now lies in the cone's upper third. Due to the energy imparted to the sand by rotating the hourglass, it becomes active and is able to create horizontal forces. If the sand were given free room, it would return to its lying wedge shape.


Fig. 1 shows unequal soil bodies and earth forces in an hourglass.

Consequently, there are two different calculation methods for force determination, which are connected with the location of the respective center of gravity. To distinguish them, the inactive wedge shape will be called 'lying' and wedge standing on its head will be called 'standing'. In both wedge shapes, horizontal forces are generated purely by the soil weight resting on the friction/inclination plane under inclination angle 6. This reveals that no horizontal earth stresses occur in the soil body's basal plane - whether lying or standing - as specified in the rules and standards of the current teachings.

### 2.2 Calculability of soil properties

Another important innovation for earth pressure teachings is the calculability of soil properties. Hereby, it is irrelevant whether the soils are in the dry, moist or wet state, or are 'under water' (groundwater). By means of their friction/inclination angle 8 , all soil types can be classified steplessly in the 'semicircle of soil types'. Also the classification according to soil types, homogeneous areas, and geological naming of
soils (DIN 4023) can be dispensed with, as well as the previous use of empiric soil densities, angles, and other soil characteristics.

The calculation of soil properties such as density, friction angle, and load bearing capacity is based on the recognition that soils are decomposition products of different solid rock types. Special considerations must be given to soils with organic and metallic admixtures.
Due to the different amounts of water absorbed, the properties of soils in free nature can vary widely. By removing the water from the soil, one obtains a dry mass, which is easier to analyse. If an intact soil sample is weighed before and after drying, the amount of water absorbed by the soil is known, and therefore also the weight of the pore structure with embedded solid particles, i.e. the soil's dry density. From this, it can be derived that every soil type consists of a solid material portion Vf and a pore portion VI, whereby only the pore portion can absorb water $V w$. Furthermore, it is observable that the solids determine the dry density and volume $V f_{1}+V I=V p$ of the total volume.
Due to the plausible decay of rocky ground, it can be assumed that the solids volume $V f_{1}$ is part of the rocky ground. If one takes hard granite with its density of $p t g_{90}=3,0$ $\mathrm{t} / \mathrm{m}^{3} \rightarrow \gamma=29,4 \mathrm{kN} / \mathrm{m}^{3}$ as the limit value for all solids, this rock type will take top position in the scale of soil types. Based on their density, other original rocks must be classified below that of granite.

The conversion of granite rock into a decomposition product/soil type is understandable, if one takes granite as pore-free, and equates its solids volume $V f$ with the cube volume $V p=V f=1,00 \mathrm{~m}^{3}$. In the example, the granite must be crushed so far that a volume increase in the amount of width $\Delta b=0,70 \mathrm{~m}$ occurs in the cube. Accordingly, the new product's properties can be determined via standardization (see Fig. 3).



Fig. 2


Fig. 3
Fig. 2 shows the solids volume $V f_{90}=1,00 \mathrm{~m}^{3}$ and the pore volume $V I=0,70 \mathrm{~m}^{3}$.
Fig. 3 shows the volume of the new product after standardization.

Soil in the dry state:
Solids volume $V f^{\prime}=V f_{90} /(b b \cdot h \cdot a)=1,00 /(1,70 \cdot 1,00 \cdot 1,00)=0,588 \mathrm{~m}^{3}$
Pore volume VI' $=V p-V f^{\prime}=1,00-0,588=0,412 \mathrm{~m}^{3}$
Material density $p t g^{\prime}=V f^{\prime} \cdot p t g_{90}=0,588 \cdot 3,000=1,764 \mathrm{t} / \mathrm{m}^{3}$

It was also found that the ratio of solids volume to pore volume corresponds to the tangent of friction angle/inclination angle $\beta$. Because an inclination angle $\beta_{90}=90^{\circ}$ must be applied for pore-free granite, the angle for the crushed material is calculated as follows:

Inclination angle $\beta \rightarrow \tan V f / V I=0,588 / 0,412=1,427 \rightarrow$ angle $\beta=55,0^{\circ}$
As shown above, hard granite takes the top position in the scale of soil types. The decomposition product at the end of the scale is described as 'primordial dust'. Its ratio $V f / V I=0,01 / 0,99=0,010$ creates angle $\beta=0,58^{\circ} \sim 0,6^{\circ}$. The primordial dust's dry density is then calculated from the solids volume $V f=0,01 \mathrm{~m}^{3}$, resulting in dry density ptg $=0,01 \cdot 3,00=0,030 \mathrm{t} / \mathrm{m}^{3}$.

## Note:

For calculation reasons, the new earth pressure teachings use the dimension $t / \mathrm{m}^{3}$ for soil densities. The multiplication of density ptg (pig, png) with gravitational force $\mathrm{g}=9,807 \mathrm{~m} / \mathrm{s}^{2}$ is done subsequently when determining the force.

### 2.3 Properties of wet and moist soils

If a dry soil absorbs water, its density and its inclination angle are changed. While soil density increases, the inclination angle $\beta t$ flattens and changes to angle $\beta i$ or $\beta n$.
Opposed to dry soil is the wet soil ( n ), whose pore structure can be filled completely with water. This changes volume VI into volume VIn. Via the amount of absorbed water, moist soils (i) are aligned between the poles with volume VII. In order to introduce water volume $V w$ into the angle calculation, the fictive solids volume $V f n=$ VIn $\cdot p w g / p t g$ or $V I / 3$ is required, whereby water density is specified with $p w g=$ $1,00 \mathrm{t} / \mathrm{m}^{3}$ or $\operatorname{ptg}_{90} / 3$.

Using the example of dry soil with solids volume $V f=0,588 \mathrm{~m}^{3}$, density $p t g^{\prime}=1,764$ $\mathrm{t} / \mathrm{m}^{3}$, and angle $\beta t=55,0^{\circ}$ the changes to the characteristic values due to complete filling of the pores $V I=V n=0,412 \mathrm{~m}^{3}$ with water are shown. The fictive solids volume is $V f n=V I / 3=0,412 / 3=0,137 \mathrm{~m}^{3}$. The volumes are shown in Figs. 4 and 5 .

Wet density $p n g=p t g+V n \cdot p w g=1,764+0,412 \cdot 1,000=2,176 \mathrm{t} / \mathrm{m}^{3}$
Inclination angle $\beta n \rightarrow \tan V f /(V I+V I / 3)=0,588 /(0,412+0,412 / 3)=1,070$
Inclination angle $\beta n=47,0^{\circ}$


Fig. 4 shows the expansion of the soil body in Fig. 5 by volume Vfn of the water and the formation of inclination angle $\beta n$.
Fig. 5 shows the expansion of the soil cube by fictive volume $V f n$.

Shown in Fig. 6 are the changes to densities, angles, and forces due to complete filling of the soil pores with water. A series of tests carried out with different soils confirmed the validity of the calculated soil properties (see book, Chapter 3).


Fig. 6 shows the increase of earth pressure force due to water absorbance.

### 2.4 Earth block, its forces and their distribution

To enable soils in free nature to create forces, corresponding free spaces are required. In order to make these spaces recognizable, a fixed value is necessary, which was found the volume of an 'earth block'. If one applies the inclination plane as an area diagonal of the earth block, height $h$ and width $b$ of the block can be calculated via inclination angle $B$, soil type, and calculation depth $a=1,00 \mathrm{~m}$. The diagonal also divides block area $A=b \cdot h$ into the upper active area $A o$ and the lower reactive area $A u$ (see Fig. 7).

Fig. 8 shows the forces and their locations, as calculated according to the 'physical level'. If one compares the build-up of horizontal force $H f$ in the 'standing' earth wedge C-A-B, which is generated by the soil weight above the inclination plane, the horizontal force $L h$ in the 'lying' wedge C-B-D can only be generated under the same condition. Consequently, force H of the lying earth wedge is generated from the soil weight resting on the inclination plane with angle $b$.

Fig. 7


Fig. 8



Fig. 7 shows wedge areas $A o$ and $A u$ above and below the inclination plane.
Fig. 8 shows the location of earth pressure force in the 'lying' and 'standing' earth wedges.

For force determination, the new earth pressure teaching resorts to Monsieur de Coulomb's (1736-1806) "Classical earth pressure teachings", which he illustrates in his sketch sheet (see book, Section 2.2.).

In his Fig. 7, Coulomb places the weight force of soil volume $V=A o \cdot a$ behind the supporting wall, and calculates the individual forces as follows:
Weight force $G=A o \cdot a \cdot p t g \cdot g \quad$ (in analogy: pig, png etc.)
Normal force $F N=G e \cdot \cos B$
Downhill force $F H=G e \cdot \sin B$
Earth pressure force $F a=H f=G \cdot \sin B \cdot \cos B$.
Thrust height $h v$ of earth pressure force $H f$ against the wall is calculated as:
Height $h v=h \cdot \sin ^{2} b$
Forces can be converted into force meters, and applied to scale. For this, the 'force index' $g i$ ( $g i t, g i n$ ) is required as quotient.

Force index $g i=a \cdot b \cdot p t g \cdot \mathrm{~g} / 2$ in $\mathrm{kN} / \mathrm{m}^{2}$ (in 'standing' earth wedge)
Force meter $h=G / g i$ in meters $\quad$ Force meter $n v=N v / g i$
Force meter $f n=F N / g i \quad$ Force meter $h v=H v / g i$
Force meter $f h=F H / g i \quad$ Force meter $h f=-h n=H f / g i$
The following forces and force meters were calculated for a soil type with density $p t g^{\prime}=1,764 \mathrm{t} / \mathrm{m}^{3}$ and angle $\beta t=55,0^{\circ}$ :


Fig. 9 shows lying earth wedge at left of axis $A-B$, and the standing earth wedge at right. Fig. 10 shows the force distribution in the earth wedges (angles $\alpha=58^{\circ}$ and $B=58^{\circ}$ ).

From the figures above, it can be derived that every force in the wedge occupies a portion of wedge area Ao. Consequently, the force values of the 'lying' wedge C-B-D can be determined from area $A u^{\prime}=h \cdot b u / 4$. The calculation method above the 'physical plane' is applicable with all inclination angles $B t=0,6^{\circ}$ up to $89,4^{\circ}$. Only the forces $F N$ and $F H$ change their positions in the earth wedge:

- Angle $B>45^{\circ}$ : Normal force FN is smaller than downhill force FH.
- Angle $B=45^{\circ}$ : Normal force $F N$ and downhill force FH are equal.
- Angle $B<45^{\circ}$ : Normal force FN is larger than downhill force FH.


### 2.5 Semicircle of soil types

In the previous sections it was shown that the properties of soils and the earth forces are closely related. If one value changes, all the other values also change. This knowledge can be used for all soil types, regardless of whether they are in the dry, moist or wet state or are located under water. Consequently, it is possible to determine their earth pressure force and their thrust height via the 'semicircle of soil types'.

For this, a coordinate system is used, whose ordinates correspond to a $10,0 \mathrm{~m}$ high wall, and the earth pressure force of the respective soil type can be determined on the abscissa. Radius $r=5,00 \mathrm{~m}$ is determined by the soil type with inclination plane $B=45^{\circ}$. By placing the inclined planes at the zero point, intersections with the semicircle are created, which - referred to the respective soil type - limit the lengths of the downhill planes. The horizontal line drawn from the intersection to the ordinate then corresponds to the force meter of the earth pressure force. Force Hf is calculated by force meter times force index $g i$.
For soil types with angles $b=65^{\circ}$ and $55^{\circ}$, Fig. 11 shows the force meters of earth pressures as well as their thrust heights against the wall. In the following, earth pressure force $H f$ and its thrust height $h v$ against the wall will be calculated for a soil type with density $p t g^{\prime}=1,764 \mathrm{t} / \mathrm{m}^{3}$ and angle $B=55^{\circ}$.


Fig. 11 shows the dependencies of inclination angles and earth pressure forces, and their thrust heights against the wall in the 'semicircle of soil type'.

Wedge width $b=h / \tan B=10,0 / 1,428=7,00 \mathrm{~m}$
Force meter of earth pressure force $h f=h \cdot \sin B \cdot \cos B=10,0 \cdot 0,819 \cdot 0,574=4,70 \mathrm{~m}$ Thrust height of earth pressure force $h v=h \cdot \sin ^{2} B=10,0 \cdot 0,671=6,71 \mathrm{~m}$ Force index $g i=a \cdot b \cdot p t g \cdot g / 2=1,0 \cdot 7,00 \cdot 1,764 \cdot 9,807 / 2=121,1 \mathrm{kN} / \mathrm{m}^{2}$ Earth pressure force $H f=h f \cdot g i=4,70 \cdot 121,1=569,2 \mathrm{kN}$

If several earth blocks of the same soil type are placed next to each other, their forces permit an equilibrium in the soil/mantle to be demonstrated. Only natural or artificial interventions into the force system can cause soils to move.


Fig. 12 shows opposing earth wedges, whose forces maintain the mantle's equilibrium.

### 2.6 Load bearing capacity of soils

Determination of the load bearing capacity of soils is closely connected with the calculation of soil properties. Here, the knowledge of construction engineers is obeyed, that every type of rock is able to support a square column of the same type of rock with height $h^{*}=100 \mathrm{~m}$ without impressions appearing in the rock. If this assumption is applied to granite, a granite massif must be able to support the weight force $G=h \cdot A d \cdot \operatorname{ptg}_{90} \cdot \mathrm{~g}=100 \cdot 1,0 \cdot 3,00 \cdot 9,708=2912,4 \mathrm{kN}$ above its contact area $A d=b \cdot a=1,00 \mathrm{~m}^{2}$ :

Hence: $\sigma_{D z u l}=G / A d=2912,4 / 1,00=2.912 \mathrm{kN} / \mathrm{m}^{2}$
If the weight force $G$ is applied to a 100 m high rock column, and a one-sided direction of force is permitted, the friction plane under the tangent $\tan B t=h^{*} / b=100 / 1,0=$ 100 (friction coefficient $\mu=100$ ), and equal to inclination angle $B t=89,4^{\circ}$, will be developed. Conversely, height $h$ and width $b$ of area $A=V / a=100 \mathrm{~m}^{2}$ can be calculated, if the inclination plane under its angle (here $\beta t=55,0^{\circ}$ ) intersects area $A$ as a diagonal (see Fig. 14).

Height $h=v(A \cdot \tan B t)=v 100 \cdot \tan 55^{\circ}=11,95 \mathrm{~m}$
Width $b=\vee(A / \tan B t)=V 100 / \tan 55^{\circ}=8,37 \mathrm{~m}$
If a soil column with density $p t g^{\prime}=1,764 \mathrm{t} / \mathrm{m}^{3}$ and height $h=11,95 \mathrm{~m}$ is placed on the contact area $A d=1,00 \mathrm{~m}^{2}$, the soil type's permissible load bearing capacity can be calculated from its weight force.

Soil pressure $\sigma_{\text {Dzul } 55}=A d \cdot h \cdot \operatorname{ptg}_{55} \cdot \mathrm{~g}=1,00 \cdot 11,95 \cdot 1,764 \cdot 9,807=$ Soil pressure $\sigma_{\text {Dzul }} 55=206,7 \mathrm{kN} / \mathrm{m}^{2}$


Fig. 13 shows the vertical section through the rock column with the inclination plane (green) and the active and reactive force areas $A a$ and $A r$.
Figs. 14 and 15 shows the transition from the rock column to the soil body under the inclination angle $8 t=55,0^{\circ}$ and the inclination plane (cyan).

This calculation method for load bearing capacity can be used for all soil types. Although the weight force increases for soils in the moist and wet state, only the load bearing capacity of a dry soil should also be applied here. The lower load bearing capacity is justified, because water under pressure will give way, and therefore cannot be used for force dispersal. The soil area in which force dispersal is carried out, can be determined via the inclination angle $B i$ or $8 n$.

### 2.7 General notes on the dispersal of external forces in the soil

The load bearing capacity of soils is oriented along the permissible soil pressure $\sigma_{\text {Dzul }}$. and is influenced by the number of permitted force directions within the loaddispersing soil. Per force direction, one earth block is availabe for force dispersal, i.e. strip foundations disperse the load via two earth blocks, and single foundations via four earth blocks.


Fig. 16


Fig. 17


Fig. 18

Fig. 16 shows the two-sided force dispersal in the soil under strip foundations Figs. 17 and 18 show multi-sided force dispersal in the soil under single foundations

Fig. 19 is a photo from the Deutsche Forschungsgesellschaft für Bodenmechanik (Degebo), and shows the traces of four-sided force dispersal. This force dispersal was modified and included in Fig. 18.


Fig. 19
The volume of the soil's own weight is formed below load area Ad. Its weight force must be added to the load as the soil's own weight, and therefore be dispersed into the soil as the total load via the four earth blocks. As shown in Figs. 20 and 21, the volume of the soil's own weight can adopt the shape of a circular cone or pyramid 'standing on its point'. Height ho of the active soil body is calculated from the soil's inclination angle 8 . Within the four blocks, the volume of the soil's own weight
occupies one third of the total volume. On the surface, the areas $A k$ mark the extensions of the force dispersals.


Fig. 20 shows a circular foundation with radial force dispersal over area $A k$.
Fig. 21 shows a single square foundation with four-sided force dispersal.

Calculation examples of force dispersal under foundations are given in Sect. 3.1ff.

### 2.8 Natural shear plane of soils

In order to examine the shear plane positions of soils in lying earth wedges more closely, sand was loosely filled into the left-hand chamber of the glass container up to height $h$, and the sand surface smoothed. Following the abrupt removal of the separating glass pane, the sand slipped down into the right-hand chamber. Hereby, a natural shear plane was formed with angle $s$ ( $s=$ new designation). The sand did not loosen during slipping, so that dispersed and filled quantities remained equal. The shear plane divided the filling height $h$ into $h / 2$, and thereby showed that the double tangent of the shear angle corresponds to the natural friction angle, consequently: $\tan s=\tan B / 2$.


Fig. 22 shows the inclination plane (red) and the shear plane (green).
Additional tests with soils in dry, moist, and wet states as well as soils under water confirm that the inclination angle stands in a direct relationship to soil density. This shows that soil angle and soil density can be calculated - if one value is changed, all the other values also change, which permits a new soil type to be created.

Consequently, the conviction grows that empiric soil characteristics can only lead to faulty calculation results. The following figures show the positions of natural inclined plane, shear plane, and earth pressure force according to 'inclined plane'.


Fig. 23 shows an earth wedge standing on its point with inclination angle 8 .
Fig. 24 shows a lying earth wedge with natural shear plane and shear angle $s$.
Fig. 25 shows the arrangement of the natural soil angles in an earth wedge.

### 2.9 Horizontal forces in soils

Current teachings support the doctrine that horizontal forces cannot build up in soils in a state of rest. As proof, soil behaviour is equated to that of spheres, stating that their weight force $G$ can only be dispersed vertically into lower layers.


G


Pict. P02.20 shows that in a heap of spheres with horizontal support, force dispersal is only possible in the vertical direction.
This thesis can be disproved by placing the spheres in a container, Fig. 26.

If spheres are filled into a container, they are also in the state of rest, whereby - similar to soils - they create horizontal forces against the container walls. Therefore, movements of the supporting wall to mobilize horizontal soil stresses are superfluous.
The real dispersal of forces from sphere to sphere is shown in detail, whereby diagonal, horizontal, and vertical force directions are shown (see Fig. 27).

Because soils support themselves via the grain structure in the same way, the force paths shown for spheres can also be applied for soils.


Fig. 27 Load dispersal against a solid wall

### 2.10 Test: Determining the height level of earth pressure force

The current standards specify the same for all soil types: The greatest horizontal stress $\sigma_{x x}$ lies in the basal plane of the earth wedge (Pict. P05.120). In Mohr's stress circle (Pict. I01.70), the horizontal stress is located in the plane (Pol-Z). For wall dimensioning, this is applied at height $\mathrm{H} / 3$.


Pict. P05.120


The following test setup was used to investigate the true soil behaviour after removing the lateral support. In particular, the aim was to determine at which height the greatest earth pressure force acts on the wall to be dimensioned. For the test, five layers of dry basalt grit were built up in the left chamber of a glass container, covered by a top layer of wet basalt grit. Paper strips were inserted between the layers for the purpose of visualizing the soil behaviour after removing the separating glass pane (see Fig. 28).

After removal of the separating glass pane, a slightly concave shear plane was formed, which changed to a linear plane after removing the upper paper strip. Apart from that, the test showed that the basalt grit only moved above the natural inclined plane, i.e. in the area of the 'standing earth wedge'. No signs of movement were apparent below the inclination plane (red), which would permit conclusions to be drawn about earth forces or earth stresses $\sigma_{x x}$ in the area of the basal plane (container floor).


Fig. 28 shows the basalt grit layers in the left chamber of the glass container.
Fig. 29 shows the slipped basalt grit with formation of a slightly concave surface.


Fig. 30 shows the newly formed linear shear plane of the grit after removal of the upper paper strip.

The test shows that the earth pressure force (horizontal stress $\sigma_{x x}$ ) described in the standards does not occur in the basal plane/container floor.

### 2.11 Wall friction forces and silo theory

Current teachings see vertical frictional forces between the rear supporting wall surface and the soil behind the wall. Regarding wall friction, the teachings cite the following in the document 'Earth pressure P', page 11:


1. On the rear side of a rough wall, thrust forces occur which can influence earth pressure inclination angle $\delta_{a}$ and earth pressure force $E_{a}$.
2. With positive wall friction, a convex fracture plane is created - with negative wall friction, the curvature is concave (see Pict. P05.60 at left).

To substantiate this assumption, current teachings make use of the 'silo theory'. Here, frictional forces occur on the silo walls when the filling material is discharged.

No mention of the silo theory is found in the pertinent reference books on physics.

In his "Handbuch der Physik" (ISBN 3-446-21760-6), Horst Kuchling describes the frictional force $F_{R}$ under 7.14 as follows:
"Apart from the resistance of the surrounding medium, friction appears in case of movement as an energy-consuming resistance. It acts on the contact surfaces of two touching solid bodies and constrains the relative movement between the two bodies. Frictional force always acts parallel to the contact surface and in opposition to the movement and therefore also to the force causing the movement. Frictional force is less than the normal force or the vertical weight force. The frictional force corresponds to the downhill force".

Consequently, frictional forces between wall and soil can only be generated, if wall or soil move vertically. From the static point of view, both possibilities are excluded. On the search for frictional forces, the filling of a silo with pellets and subsequent removal of the pellets was observed in several phases.


Fig. 31 shows a silo after being filled with pellets.
Fig. 32 shows the silo after partial removal of the pellets with an even top surface.
Fig. 33 shows the formation of a hollow cone and thereby the pellets' drive towards the silo's center.

Initially, the pellets form a heaped cone, whose surface line corresponds to the shear plane of the filling media (Fig. 31). After partial removal of the filling, a more or less horizontal surface develops, which gradually changes to a central, funnel-shaped depression (see Fig. 32). Through further removal of material, the depression changes to a hollow cone, whereby the pellets move from the silo wall towards the silo's center (see Fig. 33). As a result, there were insufficient pellets in contact with the silo wall to generate friction against the wall. Similar behaviour was observed with water and loose, dry sand, if these substances were filled into a funnel and were discharged via the funnel's lower opening.

Based on these observations, forces must be excluded that could generate friction on the vertical silo wall by means of the filling media.

### 2.12 Tracing external forces (loads) in soils

For the dispersal of force $P$ in his Figure 7, Coulomb supplements the area of the soil's own weight $\mathrm{C}-\mathrm{a}-\mathrm{B}$ with the area ( $\mathrm{a}-\mathrm{a}^{\prime}-\mathrm{B}^{\prime}-\mathrm{B}$ ). Hereby, Coulomb oversees that the dispersal of force $P$ is carried out via active and reactive surface portions. Consequently, the force area ( $a-a^{\prime}-B^{\prime}-B$ ) must be divided diagonally, whereby a reduced wedge width $b(\mathrm{C}-\mathrm{a})$ results for the same height $h$ (C-B'.


The division of area ( $a-a^{\prime}-B^{\prime}-B$ ) forms a steeper inclination angle $b e$ and lengthens the friction/inclined plane.

As shown Figure 7, force dispersal in the soil is usually vertical. If a barrier layer (rock or concrete) obstructs the vertical force dispersal, it continues in the horizontal direction.
Tests conducted by Degebo with sand in a concrete basin exhibited a horizontal pressure generation in the sand (see Fig. 35).

Fig. 34, Coulomb's Figure 7


The visible traces in the sand show a dispersal of the pressure force in the horizontal direction, and the formation of new friction planes under angle $B e$.
Moreover, Fig. 35 shows that the forces can be calculated by means of the physical plane.

When calculating the earth pressure of loads/external forces that are applied to soils, it must be examined in advance, in which direction force dispersal can take place in the soil.


Fig. 36 shows the inclined planes without/with load and vertical force dispersal.
Fig. 37 shows a more horizontal force dispersal due to a rock or concrete layer.

To calculate load dispersal, the load/force above height he must first be adapted to the properties of the in-situ soil. Height he is calculated by dividing force (kN) through soil density $\gamma\left(\mathrm{kN} / \mathrm{m}^{3}\right)$. If nothing else is specified, the load above the entire wedge width $b$ must be applied. Also here, the calculation depth $a=100 \mathrm{~m}$ applies.

### 2.13 Unequal vectorial soil stresses

Different from Coulomb's 'Classical Earth Pressure Teachings', where the angles of the normal and downhill force planes are directly related, the current teachings see different stresses due to unequal angles $\vartheta_{1}$ and $\vartheta_{2}$. They assign the normal stress component $\sigma_{n}$ and the stress component $\tau_{n}$ to the angles, and derive the stress difference from the efficiency factor of a Rankine steam engine. It is stated that particularly with solid bodies under pressure, different vectors are detectable, as shown below in Pict. 101.60 and Pict. P05.10.


Pict. 101.60


Pict. P05.10

The formation of unequal vectors in a solid body was to be verified with a concrete test cube. After the application of pressure, only equally long, unidirectional, and mirror-image force paths could be found in the cube.


Fig. 38 shows the fracture paths in the concrete test cube.


Fig. 39 only shows symmetrical fracture and force paths in the cube.


Fig. 40 shows the action and reaction areas in the cube.

Furthermore, it was observed that dispersal of the pressure force runs vertically in the upper and lower areas of the cube, and any remaining force is dispersed hori-
zontally only in the central area. These forces caused the concrete cube to burst, and formed the symmetrical fracture image in the cube.

The following can be derived from the concrete cube under pressure:

- Unequal vectors, as presented in Picts. 101.60 and P05.10 did nor occur in the concrete cube.
- The pressure force that could not be dispersed via the concrete's frictional resistance was converted into horizontal forces.
- The horizontal forces break up the concrete's structure, increase the pore formation in the concrete cube, and reduce its volume-related density.


### 2.14 Earth blocks and their grouping

When determining forces according to the new procedure, limits must be observed to exclude overloads in the soil. These are specified by the maximum volume of an earth block $V o=100 \mathrm{~m}^{3}$, which is derived from the 100 m high rock column that was placed on contact area $A d=1,00 \mathrm{~m}^{2}$ to determine the soil properties. Earth blocks can be grouped to determine the earth forces for dispersing loads/external forces as well as the earth forces against structures. Hereby, the respective force build-up and dispersal is carried out via active and reactive force areas and/or volume, taking the real force directions into account.


Fig. 41 shows a group of four earth blocks for load dispersal under a single foundation.
Fig. 42 shows eight earth blocks with vertical reference axis for force dispersal of pile loads. Fig. 43 shows four earth blocks with horizontal reference axis to determine forces in pipes.

## 3 Earth pressure - more detailed calculation examples

### 3.1 Load bearing capacity of a rectangular foundation

A rectangular foundation with load area $A d^{\prime}=a f \cdot b f=1,50 \cdot 2,00=3,00 \mathrm{~m}^{2}$ is specified. The adjacent soil must be in the moist state, consisting of dry density $p t g=1,800 \mathrm{t} / \mathrm{m}^{3}$ and 200 liters $/ \mathrm{m}^{3}$ water.


To be calculated are the foundation's load bearing capacity and the force dispersal in the soil of the same soil type:

1. in the dry state
2. in the moist state

Due to force dispersal, the surface will exhibit more or less oval areas (Fig. 19). To simplify the calculation system, a force dispersal using areas $2(A k 1+A k 3)$ plus $2(A k 2+A k 4)$ is selected (see Fig. 44).

Load bearing capacity of dry soil
Assuming a dry density of $p t g=1,800 \mathrm{t} / \mathrm{m}^{3}$, all other properties can be calculated:
Solids volume $V f_{t}=V f_{90} \cdot p t g / p t g_{90}=1,00 \cdot 1,800 / 3,0=0,600 \mathrm{~m}^{3}$
Pore volume $V I_{t}=V p_{90}-V f_{t}=1,000-0,600=0,400 \mathrm{~m}^{3}$
Inclination angle $B t \rightarrow \tan B t=V f_{t} / V I_{t}=0,600 / 0,400=1,500 \rightarrow B t=56,3^{\circ}$
Weight force $G t$ determined via $A d^{\prime}=3,00 \mathrm{~m}^{2} \rightarrow$ followed by:
Volume of soil column $V^{*}=A d \cdot h=100 \mathrm{~m}^{3}$
Height $h_{t} \rightarrow$ above angle $B t=56,3^{\circ}$
$h_{t}=V\left(V^{*} \cdot \tan B t / a\right)=V(100 \cdot 1,500 / 1,0)=12,25 \mathrm{~m}$
Width $b_{t}=\vee V^{*} /(\tan B t \cdot a)=\vee 100 /(1,500 \cdot 1,0)=8,16 \mathrm{~m}$
$G t=V t^{\prime} \cdot p t g \cdot \mathrm{~g}=12,24 \cdot 1,800 \cdot 9,807=216,1 \mathrm{kN}$
Soil pressure $\sigma_{D z u l}=G t / A d=293,7 / 1,00=216,1 \mathrm{kN} / \mathrm{m}^{2}$
Weight force $G t^{\prime} \rightarrow A d^{\prime} \cdot h_{t} \cdot p t g \cdot \mathrm{~g}=3,00 \cdot 12,25 \cdot 1,800 \cdot 9,807=648,7 \mathrm{kN}$

For load dispersal in the soil, the total volume $\Sigma V t$ must be determined from the load volume ( $3 \cdot V t^{\prime}$ ) and the volume of the soil's own weight $V g_{t}$. Volume $V g_{t}$ of the soil's own weight corresponds to an inverted pyramid, whose height hot is calculated from half the width of load area $A d^{\prime}=3,00 \mathrm{~m}^{2}$, and angle $B t=56,3^{\circ}(\tan B t=1,500)$.
Width $a o=a f / 2=1,50 / 2=0,75 \mathrm{~m}$
Height $h o_{t}=a o \cdot \tan B t=0,75 \cdot 1,500=1,13 \mathrm{~m}$
Volume $V g_{t}=h o_{t} \cdot A d^{\prime} / 3=1,13 \cdot 3,00 / 3=1,13 \mathrm{~m}^{3}$

Total volume $\Sigma V t=3 V i^{\prime}+V g_{t}=3 \cdot 12,24+1,13=37,85 \mathrm{~m}^{3}$
First, the area of the total volume with total height $h l_{t}=h_{t}+h o_{t}=12,24+1,13=12,37$ m is divided into the vertical soil columns $2 \cdot A k 1$ and $2 \cdot A k 2$. Subsequently, the load is dispersed via the width, i.e. via the areas $2 \cdot(A k 1+A k 3)$ and $2 \cdot(A k 2+A k 4)$. Widths $b g a$ and $b g b$ must be calculated for the force dispersal via angle $B t$ :

Area $A k 1=a f \cdot b o / 2=1,50 \cdot 0,75 / 2=0,563 \mathrm{~m}^{2}$
Volume Vk1 $=A k 1 \cdot h l_{t}=0,563 \cdot 12,37=6,96 \mathrm{~m}^{3}$

Referred to width $a f=1,50 \mathrm{~m}$ :
Height $h g a=V(V k 1 \cdot \tan B t / a f)=V(6,96 \cdot 1,500 / 1,5)=2,64 m$
Width $b g a=\vee V k 1 /(\tan B t \cdot a f)=\vee 6,96 /(1,500 \cdot 1,5)=1,86 \mathrm{~m}$
Width $b x=b g a-b o / 3=1,86-0,75 / 3=1,61 \mathrm{~m}$
Area $A k 2=a o \cdot(b f+b f-a f) / 2=0,75 \cdot(2,00+2,00-1,50) / 2=0,937 \mathrm{~m}^{2}$
Volume Vk2 = Ak2 $\cdot h l=0,937 \cdot 12,37=11,53 \mathrm{~m}^{3}$
Referred to width $b f=2,00 \mathrm{~m}$ :
Height $h g b=V(V k 2 \cdot \tan B t / b f)=V(11,53 \cdot 1,500 / 2,00)=2,94 \mathrm{~m}$
Width $b g b=\vee V k 2 /(\tan B t \cdot b f)=\vee 11,53 /(1,500 \cdot 2,00)=1,96 \mathrm{~m}$
Width $a x=b g b-a o / 3=1,96-0,75 / 3=1,71 m$

## Result for dry soil

Via the foundation with area $A d^{\prime}=3,00 \mathrm{~m}^{2}$, dry soil can support the weight force Gte $_{\text {zul. }}=\mathbf{6 4 8} \mathbf{3} \mathbf{3} \mathbf{k N}$, whereby the foundation's own weight must be subtracted. With all-sided force dispersal, the following wedge-shaped force areas will be formed in the soil:
$V k 1 \rightarrow$ width $a f=1,50 \mathrm{~m}$, height $h g a=2,64 \mathrm{~m}$ and depth (width) $b x=1,61 \mathrm{~m}$. $V k 2 \rightarrow$ width $b f=2,00 \mathrm{~m}$, height $h g b=2,94 \mathrm{~m}$ and depth (width) $a x=1,71 \mathrm{~m}$
Soil subsidence due to the load must be prevented.

## Load bearing capacity of moist soil

Volumes $V f_{t}=0,600 \mathrm{~m}^{3}$ and $V I_{t}=0,400 \mathrm{~m}^{3}$ must be used. Taking the water into account, the other values are calculated as follows:

Fictive water volume Vfn
$V f n=V I_{i} \cdot p w g / p t g_{90}=0,200 \cdot 1,0 / 3,0=0,067 \mathrm{~m}^{3}$
Inclination angle $B i$ of moist soil
$\tan B i=V f_{t} /\left(V I_{t}+V f n\right)=0,600 /(0,400+0,067)=1,285 \rightarrow B i=52,1^{\circ}$
Moist density pig $\rightarrow$ with $V f_{90}=1,00 \mathrm{~m}^{3}$
$p i g=\left(V f_{t} \cdot p t g_{90}+V l i \cdot p w g\right) / V f_{90}$
pig $=(0,600 \cdot 3,00+0,200 \cdot 1,0) / 1,0=2,00 \mathrm{t} / \mathrm{m}^{3}$
Weight force $\mathrm{Gi}: \rightarrow$ moist soil
Volume of soil column $V^{*}=A d \cdot h=1,00 \cdot 100=100 \mathrm{~m}^{3}$
Height $h_{i} \rightarrow$ via angle $B i=52,1^{\circ}$
$h_{i}=\mathrm{V}\left(V^{*} \cdot \tan B t / a\right)=V(100 \cdot 1,285 / 1,0)=11,34 \mathrm{~m}$

Width $b_{i}=\vee V^{*} /(\tan B t \cdot a)=\vee 100 /(1,285 \cdot 1,0)=8,82 \mathrm{~m}$
Volume $\mathrm{Vi}^{\prime} \rightarrow$ per force direction and contact area $A d=1,00 \mathrm{~m}^{2}$
Soil column $V i^{\prime}=A d \cdot h_{i}=1,00 \cdot 11,34=11,34 \mathrm{~m}^{3}$
Weight force $G i \rightarrow$ per soil column, using dry density ptg $=1,800 \mathrm{t} / \mathrm{m}^{3}$ and
$\mathrm{g}=9,807 \mathrm{~m} / \mathrm{s}^{2}$, therefore:
$G i=V i^{\prime} \cdot p t g \cdot g=11,34 \cdot 1,800 \cdot 9,807=200,2 \mathrm{kN}$
Soil pressure $\sigma_{D z u l}=G i / A d=200,2 / 1,00=200,1 \mathrm{kN} / \mathrm{m}^{2}$

## Result

For moist soil, the permissible weight force Gie is calculated via the soil pressure $\sigma_{D z u l}$ $=200,1 \mathrm{kN} / \mathrm{m}^{2}$ and the three specified soil columns of the foundations $A d=1,00 \mathrm{~m}^{2}$ $\rightarrow \mathrm{Ad}^{\prime}=3,00 \mathrm{~m}^{2}$, therefore weight force $\mathbf{G i e}_{\text {zul. }}=\sigma_{\text {Dzul }} \cdot 3 \cdot \mathrm{Ad}=200,1 \cdot 3 \cdot 1,00=\mathbf{6 0 0 , 3}$ kN

## Load dispersal in moist soil

For load dispersal, the total volume $\Sigma V i$ must be determined. This consists of the volume of weight force $V i^{\prime}$ and the volume of the soil's own weight $V g i i^{\prime}$, which is calculated using the shape of a pyramid.

Width $a o=a f / 2=1,50 / 2=0,75 \mathrm{~m}$
Height $h o_{i} \rightarrow$ angle $B i=52,1^{\circ}$ with tan $B i=1,285$
$h o_{i}=a o \cdot \tan B i=0,75 \cdot 1,285=0,96 \mathrm{~m}$
Volume $V g_{i}=h o_{i} \cdot A d^{\prime}=0,96 \cdot 3,00 / 3=0,96 \mathrm{~m}^{3}$
Total volume $\Sigma V i=3 V i^{\prime}+V g_{i}=3 \cdot 11,34+0,96=34,98 \mathrm{~m}^{3}$

The total volume with total height $h l=h i+h o_{i}=11,34+0,96=12,30 \mathrm{~m}$ must be divided into soil columns $2 \cdot A k 1$ and $2 \cdot A k 2$. Their depths $b g a$ and $b g b$ are calculated via angle $B i$, so that load dispersal is divided into the areas $2 \cdot(A k 1+A k 3)$ and $2 \cdot(A k 2$ $+A k 4)$ :

Area $A k 1=a f \cdot b o / 2=1,50 \cdot 0,75 / 2=0,563 \mathrm{~m}^{2}$
Volume Vk1 $=A k 1 \cdot h l=0,563 \cdot 12,30=6,92 \mathrm{~m}^{3}$

Referred to width af $=1,50 \mathrm{~m}$ :
Height $h g a=V(V k 1 \cdot \tan B i / a f)=V(6,92 \cdot 1,285 / 1,5)=2,43 \mathrm{~m}$
Depth $b g a=\vee V k 1 /(\tan B i \cdot a f)=V 6,92 /(1,285 \cdot 1,5)=1,89 \mathrm{~m}$
Depth $b x=b g a-b o / 3=1,89-0,75 / 3=1,64 \mathrm{~m}$
Area $A k 2=a o \cdot(b f+b f-a f) / 2=0,75 \cdot(2,00+2,00-1,50) / 2=0,937 \mathrm{~m}^{2}$
Volume Vk2 $=A k 2 \cdot h l=0,937 \cdot 12,30=11,43 \mathrm{~m}^{3}$
Referred to width $b f=2,00 \mathrm{~m}$ :
Height $h g b=V(V k 2 \cdot \tan B i / b f)=V(11,43 \cdot 1,285 / 2,00)=2,71 \mathrm{~m}$
Width $b g b=\vee V k 2 /(\tan B i \cdot b f)=\vee 11,43 /(1,285 \cdot 2,00)=2,11 \mathrm{~m}$
Width $a x=b g b-a o / 3=2,11-0,75 / 3=1,86 \mathrm{~m}$

## Result for moist soil

Via the foundation with area $A d^{\prime}=3,00 \mathrm{~m}^{2}$, moist soil can support the weight force $\mathbf{G i e}_{\text {zul. }}=\mathbf{6 0 0 , 3} \mathbf{~ k N}$, whereby the foundation's own weight must be subtracted. With all-sided force dispersal, the following wedge-shaped force areas will be formed in the soil:
$V k 1 \rightarrow$ width af $=1,50 \mathrm{~m}$, height $h g a=2,43 \mathrm{~m}$, and depth (width) $b x=1,64 \mathrm{~m}$.
$V k 2 \rightarrow$ width $b f=2,00 \mathrm{~m}$, height $h g b=2,71 \mathrm{~m}$, and depth (width) $a x=1,86 \mathrm{~m}$ With increasing water content, the load bearing capacity of the soil is reduced.

## Comparison of the results

The calculations show that a soil's load bearing capacity is greatly influenced by its state: dry, moist or wet. In the dry state, the selected soil can disperse the external force $\boldsymbol{G t e}_{\text {zul. }}=\mathbf{6 4 8 , 3} \mathbf{~ k N}$, and force $\boldsymbol{G i e}_{\text {zul }}=\mathbf{6 0 0 , 3} \mathbf{~ k N}$ in the moist state. This means a load bearing reduction of about $8 \%$. The foundation's own weight is not included in both forces.
Considering that the soil is able to absorb twice the amount of water, a further reduction of load bearing capacity is possible.

### 3.2 Load bearing capacity of soils with permissible subsidence

Soil subsidence, regardless of whether under foundations or piles, represents excessive ground loading. The higher force reduces the pore volume of the loaded soil, thereby changing its properties, such as inclination angle and density. This change of soil structure is not subject to any time limit, i.e. soil subsidence as well as its consequences can also occur many years later.


In the following, a strip foundation with width $b f=1,00 \mathrm{~m}$ is assumed, which has subsided by height $\Delta h=0,08 \mathrm{~m}$ due to the permissible soil pressure being exceeded. Load dispersal is shown here one-sided, but acts inversely, i.e. two-sided (see Fig. 45).

Under the foundation, a soil type with dry density $\operatorname{ptg}_{55}=1,764 \mathrm{t} / \mathrm{m}^{3}$, inclination angle $b t$ $=55,0^{\circ}$ and calculated soil pressure $\sigma_{\text {Dzul }} 55=$ $206,7 \mathrm{kN} / \mathrm{m}^{2}$ is selected. Already determined for one-sided force dispersal are height $h g=$ $11,95 \mathrm{~m}$ and width $b g=8,37 \mathrm{~m}$ of the loaddispersing force field (see Fig. 14 in Sect. 2.6.).

Fig. 45
To be determined is weight force $G e^{*}$, which has caused the foundation's subsidence as well as soil compaction.

## Force dispersal without foundation subsidence

The load required for a soil pressure of $\sigma_{\text {Dzul } 55}=206,7 \mathrm{kN} / \mathrm{m}^{2}$ consists of a $11,95 \mathrm{~m}$ high soil column with base area $A d=1,00 \mathrm{~m}^{2}\left(h=206,7 / \operatorname{ptg}_{55} \cdot \mathrm{~g}=11,95 \mathrm{~m}\right)$. For a strip foundation with bilateral force dispersal, width $b g=1,00 \mathrm{~m}$ and therefore also load volume $V=11,95 \mathrm{~m}^{3}$ must be halved. Calculated density $\operatorname{ptg}_{55}=1,764 \mathrm{t} / \mathrm{m}^{3}$.

First, volume Vo of the soil's own weight must be determined:
Height of earth wedge $h o=b g \cdot \tan 55^{\circ} / 2=1,0 \cdot 1,428 / 2=0,71 \mathrm{~m}$
Volume $V o=h o \cdot a \cdot b g / 2=0,71 \cdot 1,0 \cdot 1,0 / 2=0,355 \mathrm{~m}^{3}$
Volume of load $V^{\prime}=V \cdot b g / 2=11,95 \cdot 1,00 / 2=5,98 \mathrm{~m}^{3}$
Total volume $V^{*}=V o+V=0,355+5,98=6,34 \mathrm{~m}^{3} \rightarrow A e=V^{*} / a=6,34 \mathrm{~m}^{2}$
Height $h=h g=V(A e \cdot \tan B t)=V 6,34 \cdot \tan 55,0^{\circ}=3,00 \mathrm{~m}$
Width $b=b g=\vee(A e / \tan B)=\vee 6,34 / \tan 55,0^{\circ}=2,11 \mathrm{~m}$
Soil column with density $p t g_{55}$
Height $h^{\prime}=V(A e \cdot \tan B t)=V 5,98 \cdot \tan 55,0^{\circ}=2,92 \mathrm{~m}$
Width $b^{\prime}=\vee(A e / \tan B)=V 5,98 / \tan 55,0^{\circ}=2,05 \mathrm{~m}$
Load due to soil column on base area $A d=1,00 \mathrm{~m}^{2}$
Height of column $h g=2 \cdot h^{\prime} \cdot b^{\prime}=2 \cdot 2,92 \cdot 2,05=11,97 \mathrm{~m}$
Soil pressure $\sigma_{\text {vorh }}=h g \cdot p t g_{55} \cdot \mathrm{~g}=11,97 \cdot 1,764 \cdot 9,807=207,1 \mathrm{kN} / \mathrm{m}^{2}$

## Force dispersal with foundation subsidence by height $\boldsymbol{\Delta h}=\mathbf{0 , 0 8} \mathbf{m}$

The original height $h=11,95 \mathrm{~m}$ of the load-dispersing soil column is reduced by $\Delta h=$ $0,08 \mathrm{~m}$, resulting in height $h^{\prime}=11,95-0,08=11,87 \mathrm{~m}$. In the same way, density $p t g_{55}$ $=1,764 \mathrm{t} / \mathrm{m}^{3}$ is increased, thereby changing to density $p t g^{\prime}=p t g_{55^{\circ}} h / h^{\prime}=$ $1,764 \cdot 11,95 / 11,87=1,776 \mathrm{t} / \mathrm{m}^{3}$

Angle $8 t^{\prime}$ can be determined via the solid and pore portions $V f^{\prime}$ and $V I^{\prime}$ :
$V f^{\prime}=V p \cdot p t g^{\prime} / p t g_{90}=1,0 \cdot 1,776 / 3,00=0,592 \mathrm{~m}^{3}$
$V l^{\prime}=V p-V f^{\prime}=1,00-0,592=0,408 \mathrm{~m}^{3}$
Angle $b t^{\prime}=\rightarrow \tan B t^{\prime}=V f^{\prime} / V l^{\prime}=0,592 / 0,408=1,451 \rightarrow 6 t^{\prime}=55,4^{\circ}$
Height of earth wedge $h o=b g \cdot \tan 55,4^{\circ} / 2=1,0 \cdot 1,451 / 2=0,73 \mathrm{~m}$
Volume Vo = ho $\cdot a \cdot b g / 2=0,73 \cdot 1,0 \cdot 1,0 / 2=0,36 \mathrm{~m}^{3}$
Volume of load $V^{\prime}=\left(h^{\prime}+\Delta h\right) \cdot b g / 2=12,03 \cdot 1,00 / 2=6,01 \mathrm{~m}^{3}$
Total volume $V^{*}=V o+V^{\prime}=0,36+6,01=6,37 \mathrm{~m}^{3} \rightarrow A e=V^{*} / a=6,37 \mathrm{~m}^{2}$
Height $h=h g=V(A e \cdot \tan B t)=V 6,37 \cdot \tan 55,4^{\circ}=3,04 \mathrm{~m}$
Width $b=b g=\vee(A e / \tan B)=V 6,37 / \tan 55,4^{\circ}=2,10 \mathrm{~m}$
Soil column with density $p t g^{\prime}=1,776 \mathrm{t} / \mathrm{m}^{3}$
Height $h^{\prime}=V\left(A e^{\prime} \cdot \tan B t\right)=V 6,01 \cdot \tan 55,4^{\circ}=2,98 \mathrm{~m}$
Width $b^{\prime}=\vee\left(A e^{\prime} / \tan B\right)=V 6,01 / \tan 55,4^{\circ}=2,04 \mathrm{~m}$
Load on base area $A d=1,00 \mathrm{~m}^{2}$ due to soil column with height hg*
Height $h g^{*}=2 \cdot h^{\prime} \cdot b^{\prime}=2 \cdot 2,98 \cdot 2,04=12,16 \mathrm{~m}$
Soil pressure $\sigma_{\text {vorh }}=h g^{\prime} \cdot \operatorname{ptg}_{55,4} \cdot \mathrm{~g}=12,16 \cdot 1,776 \cdot 9,807=210,1 \mathrm{kN} / \mathrm{m}^{2}$

## Result

The calculations prove that even a slight exceedance of the permissible soil pressure from $\sigma_{\text {Dzul }} 55=206,7 \mathrm{kN} / \mathrm{m}^{2}$ to $\sigma_{\text {Dzul }} 55,4=210,1 \mathrm{kN} / \mathrm{m}^{2}$ can result in a significant soil subsidence of $\Delta h=0,08 \mathrm{~m}$. With the same soil type in the moist or wet state, subsidence would be increased.


In a similar manner as above, the load on a foundation could be increased mathematically, if the load calculation takes an existing anchoring depth (DIN 1054) into account, as shown in Fig. 46.

Using the anchoring depth to increase the load is not advisable, because excavations around the foundation at a later time are often unavoidable. In these cases, the force reserves are missing, which can easily lead to serious foundation subsidences. As shown, only a slight load increase results in a subsidence of 8 cm .

Fig. 46

### 3.3 General notes on load dispersal via piles in soils

Piles disperse their load via the pressure against the pile skin in the adjacent soil. Therefore, the size of the load depends on the horizontal earth pressure force acting on the pile. In a similar manner as for a single foundation, piles that are too closely arranged will disturb force dispersal, resulting in a reduction of pressure against the pile skin and thereby also to a limitation of the load (sagging of the pile). The force of the pile load is dispersed into the adjacent soil via eight earth blocks, arranged in two levels (see Fig. 42).


Because the load is transferred into the adjacent soil via a polydirectional force (earth pressure) against the pile skin, the surface texture/roughness of the pile skin is insignificant. The forces in the upper level are calculated via block height ho, whereby a cone-shaped force distribution into the soil is assumed. Soil density and the inclination/friction angle $b$ are determined by the adjacent soil type (see Sect. 2.6.).
Fig. 47


Fig. 48


Fig. 49 shows force dispersal of the load and of the soil's own weight.

For force determination, the system of 'lying earth wedges' must be applied. As shown in Fig. 47, forces $L h$ and $L n$ form a dual cone with height ho and radius re. Hereby, the earth pressure forces act against the vertical pile axis. If the acting force is shifted from the reference axis to the pile skin, height $h /$ and radius ree $=r+r e$ form a section plane on the axis.

The earth pressure forces on both sides of the axis must be determined. These forces must be reduced by the force amounts acting on the pile radius. Consequently, only the earth pressure forces remain, which can be used to disperse the pile load into the adjacent soil. Force $G_{f}$ at the level below the pile foot can be determined via soil pressure $\sigma_{\text {Dzul }}$ and the load area of the foot. The force is dispersed via the cone volume Vae, which includes the volume Vo of the soil's own weight (see Fig. 49).

The force dispersal corresponds to that of a circular foundation (see book, page 169, Sect. 4.3.5.). Examples will follow.

### 3.4 Single pile, integrated in the upper force level

Specified is an in-situ concrete pile with diameter $d=0,60 \mathrm{~m}$ (without widened foot) and density $p_{p f}=2,40 \mathrm{t} / \mathrm{m}^{3}$, which is to be set in a moist soil. In the dry state, the soil has the density $p t g=1,764 \mathrm{t} / \mathrm{m}^{3}$ and inclination angle $8 t=55^{\circ}$. The water absorbed by the soil's pore structure is specified at 180 liters per $1,0 \mathrm{~m}^{3}$.
To be calculated are pile height $h p$ and payload $G G^{*}$, which can be applied to the pile without overloading the soil.

## Properties of moist soil

First, solids volume Vf and pore volume VI must be determined:
Solids volume $V f=V p_{90} \cdot p t g / p t g_{90}=1,00 \cdot 1,764 / 3,00=0,588 \mathrm{~m}^{3}$
Pore volume $V I=V p_{90}-V f=1,000-0,412 \mathrm{~m}^{3}$
Moist density pig $=(0,588 \cdot 3,0+0,180 \cdot 1,0) / 1,0=1,944 \mathrm{t} / \mathrm{m}^{3}$
Pore volume $\rightarrow$ occupied by water: $\mathrm{VIn}=0,180 \mathrm{~m}^{3}$
Pore volume $\rightarrow$ not occupied by water: $V / t=V I_{55}-V I n=0,412-0,180=0,232 \mathrm{~m}^{3}$
Fictive solids volume $V f n$
$V f n=V I n \cdot p w g / p t g_{90}=0,180 \cdot 1,0 / 3,0=0,060$
Inclination angle $B i$
$\tan B i=V f /(V I+V f n)=0,588 /(0,412+0,060)=1,246 \rightarrow B i=51,2^{\circ}$

## Permissible soil pressure $\sigma_{D z u}$ and force dispersal

Soil pressure is determined via force area $A=V^{*} / a=100 \mathrm{~m}^{2}$ and inclination angle $8 i$ $=51,2^{\circ}, \tan B i=1,246$ with one-sided force dispersal:

Force area: height $h_{51}=$
$h_{51}=\vee V^{*} \cdot \tan B i / a=\vee 100 \cdot 1,246 / 1,0=11,16 \mathrm{~m}$
Force area: width $b_{51}=$
$b_{51}=\vee V^{*} / \tan B i / a=\vee 100 \cdot 1,0 / 1,246=8,96 \mathrm{~m}$
Weight force $G t \rightarrow$ with $\mathrm{g}=9,807 \mathrm{~m} / \mathrm{s}^{2}$
$G t=V i \cdot p t g_{55} \cdot \mathrm{~g}=11,16 \cdot 1,764 \cdot 9,807=193,1 \mathrm{kN}$
Soil pressure $\sigma_{D z u}=G t / A d=193,1 / 1,00=193,1 \mathrm{kN} / \mathrm{m}^{2}$

With polydirectional force dispersal, the one-sided force dispersal with height ho and width $b$ determined previously via depth $a=1,00 \mathrm{~m}$ is reduced, as is the case for a single pile. The inclination angle of the adjacent soil remains unchanged. To determine the new height $h^{\prime}$ and width $b^{\prime}$, a 100 m high soil column with volume $V^{*}$ $=100 \mathrm{~m}^{3}$ of the same soil is again placed on the square load area $A d=1,00 \mathrm{~m}^{2}$, and polydirectional force dispersal is permitted:

Height $h o=h p={ }^{3} V V^{*} \cdot \tan B i^{2}={ }^{3} V 100 \cdot 1,246^{2}=5,37 \mathrm{~m}$
Width $b^{\prime}=r e=V V^{*} / h o=V 100 / 5,37=4,31 \mathrm{~m}$
Base area $A k=b^{\prime 2}=4,31^{2}=18,6 \mathrm{~m}^{2}$

## Force acting against the pile axis with height $\boldsymbol{h l}$

As stated, four earth blocks are bundled to determine the earth pressure force against the pile skin. As a result, the square base area $\Sigma A k=4 \cdot A k=4 \cdot 18,6=74,3 \mathrm{~m}^{2}$ is available, into which a cone with radius $r e=4,31 \mathrm{~m}$ and height $h o=5,37 \mathrm{~m}$ is placed. The earth pressure force to be determined for both sides of the vertical cone axis via the volume of force $L h$, would be aligned along the axis. If one wants to deviate from this normal case, and allow the full earth pressure from area $A r=h o \cdot r e / 2$ to act against the skin, the cone with radius ree $=r e+r=4,31+0,30=4,61 \mathrm{~m}$ must be placed at height $h /$ :
$h l=r e e \cdot \tan B i=4,61 \cdot 1,246=5,74 \mathrm{~m}$.
Base area of the cone $A k r=\pi \cdot r e e^{2}=\pi \cdot 4,61^{2}=66,8 \mathrm{~m}^{2}$
Cone volume $V k r=A k r \cdot h o / 3=66,8 \cdot 5,74 / 3=127,8 \mathrm{~m}^{3}$

A double cone with height $h l=5,74 \mathrm{~m}$ and diameter $d k=4,61 \mathrm{~m}$ must be inserted into the cone. Its volume $V k a$ is calculated from:
Circular cone $V k a=\pi \cdot d k^{2} \cdot h / / 3=\pi \cdot 4,61^{2} \cdot 5,74 / 12=31,9 \mathrm{~m}^{3}$

## Determination of earth pressure against the pile axis

Weight force $G_{p f}$ can be determined via volume $V k a=31,9 \mathrm{~m}^{3}$ of the double cone with density $p i g=1,764 \mathrm{t} / \mathrm{m}^{3}$, and then divided by the two wedge sides:

Weight force $G_{p f}=V k a \cdot p i g \cdot \mathrm{~g}=31,9 \cdot 1,764 \cdot 9,807=551,8 \mathrm{kN}$
Per pile side $G_{p f} f^{\prime}=G_{p f} / 2=551,8 / 2=275,9 \mathrm{kN}$

Earth pressure $L h^{\prime}=G_{p f^{\prime}} / \tan B i=275,9 / 1,246=221,4 \mathrm{kN}$ acting against the pile axis on each side. The earth pressure $L h$, which acts against the pile skin on both sides, is calculated by means of $L h=L h^{\prime} \cdot$ re / ree $=221,4 \cdot 4,31 / 4,61=207,0 \mathrm{kN}$. In this way, the pile is able to disperse the force/load $G G$ via the two-sided application of earth pressure force $L h$ :
Dispersal via the pile skin $G G=2 \cdot L n=2 \cdot L h \cdot \tan B i=2 \cdot 207,0 \cdot 1,246=515,8 \mathrm{kN}$.

## Load dispersal via the pile foot

Force $G_{f}=\sigma_{D z u l} \cdot\left(\pi \cdot r^{2}\right)=193,1 \cdot \pi \cdot 0,30^{2}=54,6 \mathrm{kN}$ can be absorbed via the pile foot $d=0,60 \mathrm{~m}$ and soil pressure $\sigma_{D z u l}=193,1 \mathrm{kN} / \mathrm{m}$. For load dispersal in the soil, the cone of the soil's own weight must be determined first, and its volume $\mathrm{V}_{f}$ is then added to volume $\mathrm{VG}_{f}$ of force $G_{f}$ :

Volume $V_{f}=\left(\pi \cdot 0,30^{2}\right) \cdot r \cdot \tan \mathrm{Bi} / 3=0,28 \cdot 0,3 \cdot 1,246 / 3=0,035 \mathrm{~m}^{3}$
Volume $V G_{f}=G_{f} /(p i g \cdot g)=54,6 /(1,764 \cdot 9,807)=31,56 \mathrm{~m}^{3}$
Volume $\sum V_{f}=V_{f}+V G_{f}=0,035+31,56=31,6 \mathrm{~m}^{3}$
An earth cone must be formed from volume $\Sigma V_{f}$, which disperses the force $G_{f}$ :
$\Sigma V_{f}=\pi \cdot h \cdot(h / \tan B i)^{2} / 3=\pi \cdot h^{3} / 1,246^{2} \cdot 3=$
$h={ }^{3} V \Sigma V_{f} \cdot 1,246^{2} \cdot 3 / \pi={ }^{3} \mathrm{~V} 31,6 \cdot 1,483=3,60 \mathrm{~m}$
$r g=h / \tan B i=3,60 / 1,246=2,89 \mathrm{~m}$

## Own weight of the pile

Volume $V_{h p}=\pi \cdot r^{2} \cdot h p / 3=\pi \cdot 0,30^{2} \cdot 5,37 / 3=0,51 \mathrm{~m}^{3}$
Pile's own weight $G G_{f}=V_{h p} \cdot p_{p f} \cdot \mathrm{~g}=0,51 \cdot 2,40 \cdot 9,807=12,0 \mathrm{kN}$

Payload GG*, which can be supported by the pile, consists of:
Force dispersal via the pile skin $G G=515,8 \mathrm{kN}$
Force dispersal via the pile foot $G_{f}=54,6 \mathrm{kN}$
Minus the pile's own weight $G G_{f}=12,0 \mathrm{kN}$
Payload $G G^{*}=515,8+54,6-12,0=558,4 \mathrm{kN}$

## Result

In moist soil with density pig $=1,944 \mathrm{t} / \mathrm{m}^{3}$ and soil angle $B i=51,2^{\circ}$, the pile with $\phi$ $0,60 \mathrm{~m}$ and height $h=5,37 \mathrm{~m}$ can support the payload $G G^{*}=558,4 \mathrm{kN}$. With this payload, the pile will not subside.

Not taken into account in this force determination are eccentric and dynamic pile loads as well as safety-relevant factors that could influence pile dimensioning.

### 3.5 Single pile, integrated in both force levels

For this example, the data and partial results of Sect. 3.4 are taken over. Also to be determined is the force that can be transferred into the adjacent soil via the pile skin of the lower level (see Fig. 50).


Adapted to the block heights $h o=h u=$ $5,37 \mathrm{~m}$, the pile height can be increased to $h p=10,74 \mathrm{~m}$. Already determined is force $G G=515,8 \mathrm{kN}$ that can be dispersed into the soil of the upper level via the pile skin. Still to be calculated is force $G G^{\prime}=2 H v$ of the lower level, whereby the force system of a 'standing earth wedge' must be applied. The calculation is started via height $h l=5,74 \mathrm{~m}$, radius ree $=4,61 \mathrm{~m}$, and inclination angle $B i=$ $51,2^{\circ}, \tan B i=1,246$.
Fig. 50

To be calculated are the earth pressure forces Hf by means of the active soil's circular cone
Angle $=8 i=51,2^{\circ}$, $\tan B i=1,246$
Height $n v=h l \cdot \cos 8 i^{2}=5,74 \cdot 0,393=2,25 \mathrm{~m}$
Height $h v=h l \cdot \sin B i^{2}=5,74 \cdot 0,607=3,49 \mathrm{~m}$
Radius ree' at height $h v$
$r e e^{\prime}=h l \cdot \cos B i \cdot \sin B i=5,74 \cdot 0,627 \cdot 0,780=2,81 \mathrm{~m}$
$d k^{\prime}=2 r e e^{\prime}=2 \cdot 2,81=5,62 \mathrm{~m}$
Volume $V k a^{\prime}=\pi \cdot d k^{\prime 2} \cdot h l / 3=\pi \cdot 5,62^{2} \cdot 5,74 / 12=47,46 \mathrm{~m}^{3}$
Weight force $G_{p f}{ }^{\prime}=V k a^{\prime} \cdot p i g \cdot \mathrm{~g}=47,46 \cdot 1,764 \cdot 9,807=821,0 \mathrm{kN}$
On each pile side $G_{p f}{ }^{\prime}=G_{p f}{ }^{\prime} / 2=821,0 / 2=410,5 \mathrm{kN}$, whereby the earth pressure force acts against the pile axis from two sides.
$L h^{\prime}=G_{p f^{\prime}} / \tan B i=410,5 / 1,246=339,5 \mathrm{kN}$
Due to the shift from the pile axis to the pile skin, force $L h^{\prime}$ is reduced, so that $L h=L h^{\prime} \cdot r e / r e e=339,5 \cdot 4,31 / 4,61=317,4 \mathrm{kN}$.
In this way, via the earth pressure on the skin, the pile can disperse the following force/load $G G^{\prime}$ by means of two-sided application of the earth pressure force $L h$ :
$G G^{\prime}=2 \cdot L n=2 \cdot L h \cdot \tan B i=2 \cdot 317,4 \cdot 1,246=791,0 \mathrm{kN}$
plus force $G G=515,8 \mathrm{kN}$ from the upper level, minus the pile's own weight from height $h p^{\prime}=10,74 \mathrm{~m}$, therefore, $2 \cdot G G_{f}=2 \cdot 12,0=24 \mathrm{kN}$.

## Result

In moist soil with density pig=1,944 t/m ${ }^{3}$ and soil angle $B i=51,2^{\circ}$, the single pile with $\phi 0,60 \mathrm{~m}$ and height $h p=10,74 \mathrm{~m}$ can disperse the payload $G G^{*}=515,8+791,0-$ $24,0=1.283 \mathrm{kN}$.
With this payload, the pile will not subside.

### 3.6 Single pile, integrated in the extended upper force level

The extension results from the application of the adjacent soil's load area directly against the pile skin. Hereby, radius $r e$ is expanded to radius ree $=r e+r=6,85+$ $0,30=7,15 \mathrm{~m}$
With one-sided force dispersal for the selected soil type with inclination angle $8 i=$ $51,2^{\circ}$, a force area with height $h_{51}=11,16 \mathrm{~m}$ and width $b_{51}=8,96 \mathrm{~m}$ was calculated (Sect. 2.6). Based on the load distribution in single foundations, a polydirectional force dispersal in the soil was assumed, which changes height $h_{51}$ to height $h o=5,37$ m . For foundations that require load application to the adjacent soil via a flat surface, this load distribution should be correct

The square column adopts height ho and width $\mathrm{b}^{\prime}$ :
Height $h o=h p={ }^{3} V \Sigma V^{*} \cdot \tan B i^{2}={ }^{3} V 400 \cdot 1,246^{2}=8,53 \mathrm{~m}$
Width $b^{\prime}=r e=V \Sigma V^{*} / h o=V 400 / 8,53=6,85 \mathrm{~m}$
Base area $A k=b^{\prime 2}=6,85^{2}=46,92 \mathrm{~m}^{2}$


The earth cone with radius $r e=6,85 \mathrm{~m}$ and height $h p=8,53 \mathrm{~m}$ must be placed in this square column. Height $h l=$ ree . $\tan B i=7,15 \cdot 1,246=8,90 \mathrm{~m}$ can then be determined via radius ree $=r e+r=$ $6,85+0,30=7,15 \mathrm{~m}$. Volume Vkr of the active double cone can be determined via height $h l$ and radius ree/ 2 or $d k=$ ree. As a result, circular cone $V k a=\pi \cdot d k^{2} \cdot h /$ $/ 3=\pi \cdot 7,15^{2} \cdot 8,90 / 12=119,1 \mathrm{~m}^{3}$ is formed (see Fig. 51).

Fig. 51

## Determination of earth pressure against the pile axis

The earth pressure force $L h$, which acts on both sides of the pile axis, can be determined via weight force $G_{p f}$ of the double cone. For this, volume $V k a=119,1 \mathrm{~m}^{3}$ and soil density pig $=1,764 \mathrm{t} / \mathrm{m}^{3}$ must be used.

Calculation:
Weight force $G_{p f}=V k a \cdot p i g \cdot \mathrm{~g}=119,1 \cdot 1,764 \cdot 9,807=2060,0 \mathrm{kN}$,
per pile side $G_{p f}{ }^{\prime}=G_{p f} / 2=2060 / 2=1030,0 \mathrm{kN}$.
Earth pressure force against pile axis: $L h^{\prime}=G_{p f}{ }^{\prime} / \tan B i=1030,0 / 1,246=826,8 \mathrm{kN}$

For the application of earth pressure force against the pile skin, earth pressure force $L h$ ' must be reduced to force $L h$ :
Earth pressure force $L h=L h^{\prime} \cdot r e /$ ree $=826,8 \cdot 6,85 / 7,15=792,1 \mathrm{kN}$.
Via the two-sided earth pressure force $L h=792,1 \mathrm{kN}$, the pile is able to disperse the load $G G=2 \cdot L n=2 \cdot L h \cdot \tan B i=2 \cdot 792,1 \cdot 1,246=1.973,9 \mathrm{kN}$.

## Own weight of the pile

Volume $V_{h p}=\pi \cdot r^{2} \cdot h p / 3=\pi \cdot 0,30^{2} \cdot 8,53 / 3=0,80 \mathrm{~m}^{3}$
Pile's own weight $G G_{f}=V_{h p} \cdot p_{p f} \cdot \mathrm{~g}=0,80 \cdot 2,40 \cdot 9,807=18,9 \mathrm{kN}$

Payload GG*, which can be supported by the pile, consists of:
Force dispersal via the pile skin $G G=1.973,9 \mathrm{kN}$
Force dispersal via the pile foot $G_{f}=54,6 \mathrm{kN}$, as previously determined,
minus the pile's own weight $G G_{f}=18,9 \mathrm{kN}$
Payload $G G^{*}=1.973,9+54,6-18,9=2.009,6 \mathrm{kN}$

## Result

As stated above, it should be checked whether the calculation using the four square earth blocks with volume $\Sigma \mathrm{V}^{*}=400 \mathrm{~m}^{3}$ and one-sided force dispersal is suitable for the pile. In this case, a pile with $\phi 0,60 \mathrm{~m}$ and height $h=8,53 \mathrm{~m}$ in moist soil with density $p i g=1,944 \mathrm{t} / \mathrm{m}^{3}$ and soil angle $B i=51,2^{\circ}$ can disperse the payload $G G^{*}=$ 2009,6 kN.
Not taken into account in this force determination are eccentric and dynamic pile loads as well as safety-relevant factors that could influence pile dimensioning.

### 3.7 General notes on underground pipes

In the relevant specialist literature, hardly any other subject is covered as differentiatedly as the "Vertical loads on underground drain pipes". In order to provide clarification, the IKT Institute for Underground Infrastructure in Gelsenkirchen (www.ikt.de) carried out test setups in 2003 and published them in the document "Erneuerung mit Berstverfahren - Bemessung, Prüfung und Qualitätssicherung von Abwasserrohren" (Renewal with pipe bursting - Dimensioning, testing, and quality assurance for drain pipes) see Figs. 20 to 27, pages 29 ff .

Adapted to the laboratory equipment, the IKT selected a pipe covering using a somewhat sandy soil with a height of $0,70 \mathrm{~m}$. Above the pipe, the "missing" soil volumes between $2,00 \mathrm{~m}$ and $8,00 \mathrm{~m}$ were compensated by means of varying mechanical pressures. To simulate live loads, the pressure on the soil of the pipe covering was increased. With reference to the frictional forces according to the silo theory, the laboratory test results currently represent the default calculation used in ATV-DVWK A 161. The IKT Institute's conclusions must be countered with the fact that varying pressures on a pipe cannot correspond to the real earth forces that are formed in the trench backfilling under free force development. Moreover, the IKT and ATV use the silo theory to substantiate a wall friction that does not exist in this form (see Sect. 2.11).


Fig. 52 shows the force areas at left without live load, and at right with live or static load.

Fig. 43 in Sect. 2.14 shows that the new earth pressure teachings draw a vertical and a horizontal axis through the pipe's centerline, and places an earth block in every quadrant. This force system can be applied with trenchless pipe laying or mined tunnels. For pipe laying in open trenches, the earth forces of the adjacent soil are aligned with the trench wall. This causes the horizontal system axis to shift from the pipe center to the trench floor. Inclination angle 6 , height $h=h o=h u$, and width $b=b o=b u$ of the force areas can be determined via soil density, volume $V^{*}$ $=100 \mathrm{~m}^{3}$, and depth $a=1,00 \mathrm{~m}$ of the earth block.

Figs. 53 and 54 below show that the force areas of the two force levels are initially aligned along the perpendicular axis $\mathrm{A}-\mathrm{B}$. This determines the height and width of the force areas via inclination angle $B$. The maximum influence of the force on the pipe is limited, if the force area along the axis A-B reaches height $h=h g=V(A e \cdot \tan B t)$, see Figs. 14 and 15 in Sect. 2.6.
Accordingly, the force areas $A r$ and $A a$ must be taken up to the perpendicular axis G$\mathrm{G}^{\prime}$ (see Fig. 53 , right side). Fig. 54 shows the procedure, if a live load with substitute load height he is to be included in force determination. Due to the external load, the inclination angle $b$ changes to angle $b e$.
In the upper quadrants, the polar earth pressure forces against plane G-G' are calculated via the 'lying' wedge areas, and in the lower quadrants via the 'standing' wedge shapes. Calculation examples will follow.

The test series 6 was carried out to clarify the most frequent damage events in sewer construction, such as axis shifts and sags in the pipe, pipe cracks, pipe breaks, and severed connections to the buildings. It was intended to simulate the effects if an open sewer trench is filled with a material whose density far exceeds that of the adjacent soil. For the test, dry basalt grit with density $p t g=1,850 \mathrm{t} / \mathrm{m}^{3}$ was used as backfilling material, and industrial cotton wool for the adjacent soil with less loadbearing capacity. The test setup is described in the book.


Fig. 53

Fig. 55 shows that the type of trench backfilling can cause the pipe to shift from its original position, causing its connections to the buildings to tear off.


Fig. 55 shows the displacement of the lighter soil by the heavier trench backfilling.

### 3.8 Calculation example - Pipe DN 500 Sb without live load

A drain pipe DN 500 Sb is to be laid in the soil of an open shored trench with depth $h s=5,00 \mathrm{~m}$. The pipe has a with wall thickness $s=80 \mathrm{~mm}$, and is to be laid on a bedding with height $h b=0,10 \mathrm{~m}$. Panels of $v d=0,12 \mathrm{~m}$ are used to shore the trench. Working space width $b g$ is determined in accordance with the regulations. The trench soil has a moist density $p i g=1,845 \mathrm{t} / \mathrm{m}^{3}$ and a water volume $V w=0,102 \mathrm{t} / \mathrm{m}^{3}$. 'Granular soil' is to be used for trench backfilling. In the compacted state, and with a water content $V w=0,060 \mathrm{~m}^{3}$, the soil's moist density $p i g=2,120 \mathrm{t} / \mathrm{m}^{3}$. Live loads are not taken into account.

To be determined are the dimensions of the sewer trench and the forces acting on the installed pipe.


Fig. 56 shows the earth wedges of the trench backfilling at center, and those of the adjacent soil at left.

## To be calculated are:

1. Trench cross-section

Trench depth $h g=h s+s+h b=5,00+0,08+0,10=5,18 \mathrm{~m}$
Trench width $b g=d i+2(s+a r+v d)=0,50+2(0,08+0,50+0,12)=1,90 \mathrm{~m}$
Height of pipe covering $h d=h s-d i-s=5,00-0,50-0,08=4,42 \mathrm{~m}$
Outer pipe radius $r a=(r i+s)=0,25+0,08=0,33 \mathrm{~m}$
Calculation height for the adjacent soil $h=h g=5,18 \mathrm{~m}$
2. Adjacent soil: determination of the soil angles

Dry density ptg $=p i g-w=1,845-0,102=1,743 \mathrm{t} / \mathrm{m}^{3}$
Solids volume $V f=p t g / p t g_{90}=1,743 / 3,00=0,581 \mathrm{t} / \mathrm{m}^{3}$
Pore volume $V I=V p-V f=1,0-0,581=0,419 t / m^{3}$
Angle $\tan B t=V f / V I=0,581 / 0,419=1,387 \rightarrow B t=54,2^{\circ}$
Angle $\tan B i=V f /(V I+V w)=0,581 /(0,419+0,102 / 3)=1,282 \rightarrow B i=52,0^{\circ}$
3. Determination of force areas and forces of the adjacent soil
via height $h g=5,18 \mathrm{~m}$ and angle $B i=52,0^{\circ}$ :
Width $b o=h g / \tan B i=5,18 / 1,280=4,05 \mathrm{~m}$
Force area $A u\left(A^{\prime}-C^{\prime}-M\right)=h g \cdot$ bo $/ 2=5,18 \cdot 4,05 / 2=10,49 \mathrm{~m}^{2}$
Weight force $G u=A u \cdot a \cdot p i g \cdot g=10,49 \cdot 1,00 \cdot 1,845 \cdot 9,807=189,8 \mathrm{kN}$
Force index git $=G u / h g=189,8 / 5,18=36,64 \mathrm{kN} / \mathrm{m}^{2}$
Force $L v=$ git $\cdot h g \cdot a / 2=36,64 \cdot 5,18 \cdot 1,00 / 2=94,9 \mathrm{kN}$
or $L v=h g \cdot b o \cdot a \cdot p i g \cdot g / 4=5,18 \cdot 4,05 \cdot 1,00 \cdot 1,845 \cdot g / 4=94,9 \mathrm{kN}$
Earth pressure force $L h=$ git $\cdot$ bo $\cdot a / 2=36,64 \cdot 4,05 \cdot 1,00 / 2=74,2 \mathrm{kN}$

Force $L h=74,2 \mathrm{kN}$ acts against the shoring panels at height $h g / 2$. When the trench has been filled, two earth pressure forces act from the backfilling against the trench wall above and below the horizontal plane C-D' on both sides (see Fig. 56).

Initially, determination of the pipe load must be set back, because the specifications require that the trench is backfilled with 'granular soil'. Therefore, the material's density and the soil angle must be calculated first. As shown in Fig. 56, a 'lying' earth wedge with vertical force path is built up on the trench floor. On both sides of its inclined planes, force areas arise above and below the horizontal plane C-D'. Due to the overlap of the force areas, polar acting earth pressures develop within them, so that initially only vertical forces act on the pipe. Below the terrain surface, two 'standing' earth wedges with height hoo are formed, whose vertical forces $2 H v$ must be added to the initial pipe load. As shown in Fig. 57, the vertical load on the pipe is generated by the soil volume with depth $a=1,00 \mathrm{~m}$, width $d a=0,66 \mathrm{~m}$, and height $h s^{\prime}=h d+f s+h v-h o o$ or $h s^{\prime}=h d+f s-n v$.
4. Backfilling material: determination of the soil angles

Dry density ptg $=p i g-w=2,120-0,060=2,060 \mathrm{t} / \mathrm{m}^{3}$
Solids volume $V f=p t g / p t g 90=2,060 / 3,00=0,687 \mathrm{t} / \mathrm{m}^{3}$
Pore volume $V I=V p-V f=1,0-0,687=0,313 \mathrm{t} / \mathrm{m}^{3}$
Angle tan $B t=V f / V I=0,687 / 0,313=2,195 \rightarrow B t=65,5^{\circ}$
Angle $\tan B i=V f /(V I+V w)=0,687 /(0,313+0,060 / 3)=2,063 \rightarrow B i=64,1^{\circ}$
5. Determination of soil volume and force loading the pipe:

Taken over are width $b g=1,90 \mathrm{~m}$, width $d a=0,66 \mathrm{~m}$, and height $h d=4,42 \mathrm{~m}$.
Height hoo $=0,5 \cdot b g \cdot \tan B i=0,5 \cdot 1,90 \cdot 2,063=1,96 \mathrm{~m}$
Height $n v=$ hoo $\cdot \cos ^{2} B i=1,96 \cdot 0,191=0,37 \mathrm{~m}$
Height $h y=h o o \cdot \sin ^{2} B i=1,96 \cdot 0,809=1,59 \mathrm{~m}$
Height $r s=r a \cdot \sin 8 i=0,33 \cdot 0,900=0,30 \mathrm{~m}$
Height $f s=r a-r s=0,33-0,30=0,03 \mathrm{~m}$
Width $f r=r a \cdot \cos B i=0,33 \cdot 0,437=0,14 \mathrm{~m}$
Height $h s^{\prime}=h d+f s-n v=4,42+0,03-0,37=4,08 \mathrm{~m}$

Volume of pipe load
$V 1=a \cdot d a \cdot h s^{\prime}=1,0 \cdot 0,66 \cdot 4,08=2,693 \mathrm{~m}^{3}$
Pipe load due to distributed load $q / v$
$q / v=V 1 \cdot p i g \cdot \mathrm{~g} / d a=2,693 \cdot 2,120 \cdot 9,807 / 0,66=84,8 \mathrm{kN} / \mathrm{m}^{2}$
Acting on the pipe haunches are the forces $L v$
$L v^{\prime}=V 1 \cdot p i g \cdot g / 2=2,693 \cdot 2,120 \cdot 9,807 / 2=28,0 \mathrm{kN}$

Force $F V$ must be applied for pipe dimensioning.
It is calculated via vertical force $L v^{\prime}$ and angle $B i=64,1^{\circ}$, therefore
Force $F V=L v^{\prime} / \sin B i=28,0 / 0,90=31,2 \mathrm{kN}$


Fig. 57 shows the force distribution on the drain pipe.

## Result

For the DN 500 Sb pipe to be laid at a depth of $h s=5,00 \mathrm{~m}$, a shored trench with height $h g=5,18 \mathrm{~m}$ and width $b g=1,90 \mathrm{~m}$ is required. The distributed load $q / v=84,8 \mathrm{kN} / \mathrm{m}^{2}$ was calculated as pipe load by means of the soil column of the filling material with height $h s^{\prime}=4,08 \mathrm{~m}$, width $d a=0,66 \mathrm{~m}$, and depth $a$ $=1,00 \mathrm{~m}$. Therefore, force $F V=31,2 \mathrm{kN}$ is available for pipe dimensioning, which must be applied on both sides below the inclination angle $B i=64,1^{\circ}$ and against the pipe. The pipe bedding angle is $2 a=51,8^{\circ}$.

Overloading of the soil does not occur in the bedding area, as the pipe's own weight including the complete filling is less than the soil's density. Earth pressure force $L h=$ $74,2 \mathrm{kN}$ from the adjacent soil acts against the shoring panels on both sides.

### 3.9 Calculation example - Pipe DN 1800 Sb without live load

The following is specified for installing a drain in an open trench:
Trench depth $h s=5,00 \mathrm{~m}$, pipe wall thickness $s=0,18 \mathrm{~m}$, and thickness of the shoring panel $v d=0,12 \mathrm{~m}$. 'Granular soil' is to be used for trench backfilling and pipe bedding height $h b=0,25 \mathrm{~m}$. Already calculated are its moist density $\mathrm{pig}=2,120 \mathrm{t} / \mathrm{m}^{3}$ and inclination angle $B i=64,1^{\circ}$.
The adjacent substratum soil has a water content of $w=0,180 \mathrm{t} / \mathrm{m}^{3}$ and a moist density of $p i g=1,820 \mathrm{t} / \mathrm{m}^{3}$. Live loads are not taken into account.

To be determined are the dimensions of the sewer trench and the forces acting on the installed pipe.

## To be calculated are:

The descriptions and arrangement of the force areas can be taken from Fig. 58.

## 1. Trench cross-section

Trench depth $h g=h s+s+h b=5,00+0,18+0,25=5,43 \mathrm{~m}$
Trench width $b g=d i+2(s+a r+v d)=1,80+2(0,18+0,50+0,12)=3,40 \mathrm{~m}$
Height of pipe covering $h d=h s-d i-s=5,00-1,80-0,18=3,02 \mathrm{~m}$
Outer pipe radius $r a=(r i+s)=0,90+0,18=1,08 \mathrm{~m} \rightarrow d a=2,16 \mathrm{~m}$


Fig. 58 shows the pipe, embedded in the force areas of the rhombus acc. to Figs. 53 and 54.
2. Adjacent soil: Determining the soil angles

Dry density ptg $=p i g-w=1,820-0,180=1,640 \mathrm{t} / \mathrm{m}^{3}$
Solids volume $V f=p t g / p t g_{90}=1,640 / 3,00=0,547 \mathrm{t} / \mathrm{m}^{3}$
Pore volume $V I=V p-V f=1,0-0,547=0,453 t / \mathrm{m}^{3}$
Angle $\tan B t=V f / V I=0,547 / 0,453=1,207 \rightarrow B t=50,3^{\circ}$
Angle $\tan B i=V f /(V I+V w)=0,547 /(0,453+0,180 / 3)=1,066 \rightarrow B i=46,8^{\circ}$
3. Determining the force areas and forces of the adjacent soil/substratum by means of height $h g=5,43 \mathrm{~m}$ and angle $B i=46,7^{\circ}$ :
Width $b o=h g / \tan B i=5,43 / 1,066=5,09 \mathrm{~m}$
Force area $A u=h g \cdot b o / 2=5,43 \cdot 5,09 / 2=13,83 \mathrm{~m}^{2}$
Weight force $G u=A u \cdot a \cdot \mathrm{pig}^{\prime} \cdot \mathrm{g}=13,83 \cdot 1,00 \cdot 1,820 \cdot 9,807=246,8 \mathrm{kN}$ Force index git $=G u / h g=246,8 / 5,43=45,46 \mathrm{kN} / \mathrm{m}^{2}$
Force $L v=g i t \cdot h g \cdot a / 2=45,46 \cdot 5,43 \cdot 1,00 / 2=123,4 \mathrm{kN}$ or $L v=h g \cdot b o \cdot a \cdot p i g \cdot g / 4=5,43 \cdot 5,09 \cdot 1,00 \cdot 1,820 \cdot g / 4=123,3 \mathrm{kN}$ Earth pressure force $L h=$ git $\cdot$ bo $\cdot a / 2=45,46 \cdot 5,09 \cdot 1,00 / 2=115,7 \mathrm{kN}$ Force $L h=115,7 \mathrm{kN}$ acts against the shoring panels at height $h g / 2$.
4. Filling material: determination of the soil angles

The values are taken from the previous example:
Dry density ptg $=2,060 \mathrm{t} / \mathrm{m}^{3}$ and angle $B t=65,5^{\circ}(\tan B t=2,195)$
Moist density pig $=2,120 \mathrm{t} / \mathrm{m}^{3}$ and angle $B i=64,1^{\circ}(\tan B i=2,063)$
5. Determining the force areas loading the pipe

When looking at the structure of the force areas within the sewer trench in Fig. 57, changes due to the larger trench width are apparent. A certain assignment of the force areas is achievable, if one first applies the filling material's inclination plane with angle $B i$ as tangent to the pipe, and takes it up to the central axis and to the trench
wall. By means of radius $r a$ and angle $B i=64,1^{\circ}$ (tan 2,063 ), width $b m$ and then width boo as well as height hoo can be calculated. Consequently, force area $A v$ with height 2 hoo and width boo is formed, which acts against the trench wall. From this area, only the partial area with height $h h$ and width $b g^{\prime}=b o o-b b$ loads the pipe. At center, above the pipe, the area $A m=b m \cdot h s^{\prime}$ is formed, whereby height $h s^{\prime}=h d+f s$. Only force $H v$ from the earth wedge hoo $\cdot$ boo /2 acts on the pipe.

Radius $r a=1,08 \mathrm{~m}$ and angle $B i=64,1^{\circ}$ are used to calculate width $b m$ (see Fig. 57).
Height $r s=r a \cdot \sin B i=1,08 \cdot 0,900=0,97 \mathrm{~m}$
Height $f s=r a-r s=1,08-0,97=0,11 \mathrm{~m}$
Width $f r=r a \cdot \cos B i=1,08 \cdot 0,437=0,47 \mathrm{~m}$
Width $b m=2 \cdot f r=2 \cdot 0,47=0,94 \mathrm{~m}$
Width $b o o=(b g-b m) / 2=(3,40-0,94) / 2=1,23 \mathrm{~m}$
Height hoo $=b o o \cdot \tan B i=1,23 \cdot 2,06=2,53 \mathrm{~m}$
Area Aoo = boo $\cdot$ hoo $/ 2=1,23 \cdot 2,53 / 2=1,56 \mathrm{~m}^{2}$
The following force meters arise within the wedge area Aoo:
Height $n v=h o o \cdot \cos ^{2} B i=2,53 \cdot 0,191=0,48 \mathrm{~m}$
Height $h v=h o o \cdot \sin ^{2} B i=2,53 \cdot 0,809=2,05 \mathrm{~m}$
Width $h f=h o o \cdot \sin B i \cdot \cos B i=2,53 \cdot 0,900 \cdot 0,437=0,99 \mathrm{~m}$
Area Aoo must be reduced by the area of the normal force, so that:
Area $A o o^{\prime}=$ hoo $\cdot h f / 2=2,53 \cdot 0,99 / 2=1,25 \mathrm{~m}^{2}$ remains.

## Calculation of area $A v$

$A v=2 \cdot$ hoo $\cdot$ boo $/ 2=2,53 \cdot 1,23=3,11 \mathrm{~m}^{2}$,
of which only the partial area $A v^{\prime}$ acts as a load on the pipe:
Width $b b=(b g-d a) / 2=(3,40-2,16) / 2=0,62 \mathrm{~m}$
Width $b o o^{\prime}=b o o-b b=1,23-0,62=0,61 \mathrm{~m}$
Height $h h=2 \cdot\left(b o o^{\prime} \cdot \tan B i\right)=2 \cdot(0,61 \cdot 2,063)=2,52 \mathrm{~m}$
Area $A v^{\prime}=b o o^{\prime} \cdot h h / 2=0,61 \cdot 2,52 / 2=0,77 \mathrm{~m}^{2}$

## Calculation of area Am

with width $b m=0,94 \mathrm{~m}$ and height $h s^{\prime}=h d+f s=3,02+0,11=3,13 \mathrm{~m}$
Area $A m=b m \cdot h s^{\prime}=0,94 \cdot 3,13=2,94 \mathrm{~m}^{2}$
Volumes Vm and Va can be determined by means of calculation depth $a=1,00 \mathrm{~m}$ and the areas, whereby their weigh forces load the pipe at different locations.
6. Determining the pipe loads

Weight force $G m$ is determined first, which loads the pipe crown at width $b m=0,94$ m:
Weight force $G s=a \cdot A m \cdot p i g \cdot g=1,0 \cdot 2,94 \cdot 2,120 \cdot 9,807=61,2 \mathrm{kN}$

Force Gs can be used to calculate force FV, which must be applied on both sides (see Fig. 58).

Force $F V=0,5 \cdot G s / \sin B i=0,5 \cdot 61,2 / 0,90=34,0 \mathrm{kN}$
In addition, weight force $G k$ acts vertically and on both sides of the pipe haunches.
Weight force $G k=a \cdot\left(A o o^{\prime}+A v^{\prime}\right) \cdot p i g=(1,25+0,77) \cdot 2,120 \cdot 9,807=42,0 \mathrm{kN}$

## Result

For the pipe DN 1800 Sb to be installed in the trench at depth $h s=5,00 \mathrm{~m}$, a shored trench with height $h g=5,43 \mathrm{~m}$ and width $b g=3,40 \mathrm{~m}$ must be provided. The pipe is loaded in two locations by force $F V=34,0 \mathrm{kN}$ (pipe crown), and force $G k=42,0 \mathrm{kN}$ (pipe haunch).
Force dispersal under the pipe is not taken into account, because the displaced soil's weight with density pig $=2,120 \mathrm{t} / \mathrm{m}^{3}$ is higher than the pipe's own weight plus the weight of the water filling.
Force $L h=115,7 \mathrm{kN}$ was calculated from the adjacent soil, which acts against the shoring panels at height $h g / 2=2,72$.

### 3.10 Calculation example - Pipe DN 1800 Sb with live load

For the calculation, the sewer trench dimensions, the pipe dimensions, and the properties of the soil type are taken from the example in Sect. 3.8.
Trench depth $h g=5,43 \mathrm{~m}$, trench width $b g=3,40 \mathrm{~m}$ and outside pipe radius $r a=1,08$ $\mathrm{m} \rightarrow d a=2,16 \mathrm{~m}$.
Because of the live load SLW 60 to be applied, the pipe is to be bedded in concrete. $E=3,33 \mathrm{t} / \mathrm{m}^{2}(33,3 \mathrm{kN})$ is used as the substitute surface load.

## To be calculated are:

1. Load height he and angle Be' (Fig. 59)

Load height he is calculated via the dry density $p t g=2,060 \mathrm{t} / \mathrm{m}^{3}$ of the 'granular soil'. This density is selected, because the moist density's water gives way under pressure and is therefore not available for load dispersal.
Load height he $=E / p t g=3,33 / 2,060=1,62 \mathrm{~m}$

Due to the mostly rigid connection of pipe and bedding, it is assumed that the live load can only be dispersed horizontally in the area of height $h d=3,02 \mathrm{~m}$ (see Fig. 37, Sect. 2.12). Consequently, the flatter inclination angle $B e^{\prime}$ is available for force dispersal. As mentioned previously, the load does not change the soil's density.

Angle $B e$ is calculated via tangent $B e=\tan B i \cdot h d /(h d+h e)$.
Therefore, angle $B e \cdot \rightarrow \tan B e=2,063 \cdot 3,02 /(3,02+1,62)=1,343 \rightarrow B e=53,3^{\circ}$
The dimensions $r s, f s$, and $f r$ must be determined under this angle, and force $F V$ applied against the pipe.


Fig. 59 shows the drain pipe bedded in concrete, and its force areas.
2. Determination of force areas loading the pipe

Angle $B e=53,3^{\circ}(\tan B e=1,343)$
Height $r s=r a \cdot \sin B e=1,08 \cdot 0,802=0,87 \mathrm{~m}$
Height $f s=r a-r s=1,08-0,87=0,21 \mathrm{~m}$
Width $\mathrm{fr}=\mathrm{ra} \cdot \cos B i=1,08 \cdot 0,598=0,65 \mathrm{~m}$

Height $h d=h o o=3,02 \mathrm{~m}$
Width boo =hd / tan Be=3,02 / 1,343 = 2,25 m
Height $n v=h o o \cdot \cos ^{2} b e=3,02 \cdot 0,357=1,08 \mathrm{~m}$
Height $h v=h o o \cdot \sin ^{2} b e=3,02 \cdot 0,643=1,94 \mathrm{~m}$
Width $h f=h o o \cdot \sin B e \cdot \cos B i=3,02 \cdot 0,802 \cdot 0,600=1,45 \mathrm{~m}$

Area Aoo' corresponds to the area of the downhill force, therefore:
Area $A o o^{\prime}=$ hoo $\cdot h f / 2=3,02 \cdot 1,45 / 2=2,19 \mathrm{~m}^{2}$

Calculation of area $A v^{\prime}$
Height $h h=r a+r a \cdot \tan B e=1,08+1,08 \cdot 1,343=2,53 \mathrm{~m}$
Width $b b^{\prime}=h h /(\tan B e+\tan B i)=2,53 /(1,343+2,063)=0,74 m$
Height $h h o=b b^{\prime} \cdot \tan B e=0,74 \cdot 1,343=0,99 \mathrm{~m}$
Height $h h u=b b^{\prime} \cdot \tan B i=0,74 \cdot 2,063=1,53 \mathrm{~m}$
Height $h f s=h h u-r s=1,53-0,87=0,66 \mathrm{~m}$ (Fig. 57)
Width $f r^{\prime}=r a-r s / \tan B i=1,08-0,87 / 2,063=0,66 m$

Area $A v^{\prime}=h h \cdot b b^{\prime} / 2=2,53 \cdot 0,74 / 2=0,94 \mathrm{~m}^{2}$
Area $A m^{\prime}$ above the pipe:
Area $A m^{\prime}=2 \cdot(h f s+f s) \cdot f r^{\prime} / 2=(0,66+0,21) \cdot 066=0,72 \mathrm{~m}^{2}$

## 3. Determination of pipe loads

First, weight force Gs must be determined, which loads the pipe crown at width $2 \mathrm{fr}^{\prime}=$ 1,32 m:
Weight force $G s=a \cdot\left(A o o^{\prime}+A m^{\prime}+A o o^{\prime}\right) \cdot p i g \cdot g=1,0 \cdot 2,94 \cdot 2,120 \cdot 9,807=61,2$
Weight force $G s=1,0 \cdot(2,19+0,72+2,19) \cdot 2,120 \cdot 9,807=106,0 \mathrm{kN}$

Force $F V$ (which must be applied on both sides) can be calculated from force Gs.
Force $F V=0,5 \cdot G s / \sin B e=0,5 \cdot 106,0 / 0,802=66,1 \mathrm{kN}$
Weight force $G k$ loads the pipe haunches on both sides and vertically.
Weight force $G k=a \cdot A v^{\prime} \cdot p i g \cdot g=1,0 \cdot 0,94 \cdot 2,120 \cdot 9,807=19,5 \mathrm{kN}$

## Result

For the pipe DN 1800 Sb to be installed in the trench at depth $h s=5,00 \mathrm{~m}$, a shored trench with height $h g=5,43 \mathrm{~m}$ and width $b g=3,40 \mathrm{~m}$ must be provided. The pipe is loaded on two sides by force $F V=66,1 \mathrm{kN}$, which is applied in the crown area with angle $B e=53,3^{\circ}$. Moreover, the pipe haunches are each loaded vertically by force $G k$ $=10,5 \mathrm{kN}$.
Force dispersal under the pipe is not taken into account, because the displaced soil's weight with density pig $=2,120 \mathrm{t} / \mathrm{m}^{3}$ is higher than the pipe's own weight plus the weight of the water filling.
Force $L h=115,7 \mathrm{kN}$ was calculated from the adjacent soil, which acts against the shoring panels at height $h g / 2=2,72$.

### 3.11 Landslide due to changed soil properties

A slope (embankment) can begin to slide, if its soil properties are changed by a substantial water absorbance or external loads. For the following example, an embankment with height $h=5,00 \mathrm{~m}$ and width $b u=7,50 \mathrm{~m}$ (gradient ratio $1: 1,5$ ) is selected. With a water content of $V w=65 \mathrm{l} / \mathrm{m}^{3}$, the embankment soil exhibits a moist density of $p i g=1,765 \mathrm{t} / \mathrm{m}^{3}$. Rain has converted the moist soil into a 'wet' soil, and has caused it to slide. A soil is described as 'wet' when its pores are completely filled with water.

## Properties of moist soil

Dry density ptg $=p i g-V w \cdot p w g / V p_{90}$
$p t g=1,765-0,065 \cdot 1,0 / 1,0=1,700 \mathrm{t} / \mathrm{m}^{3}$
Solids volume $V f_{t}=p t g \cdot V p_{90} / p t g_{90}=1,700 \cdot 1,0 / 3,0=0,567 \mathrm{~m}^{3}$
Pore volume $V I_{t}=V p_{90}-V f_{t}=1,000-0,567=0,433 \mathrm{~m}^{3}$

Inclination angle $B t$ of dry soil:
$\tan B t=V f / V I=0,567 / 0,433=1,309 \rightarrow$ angle $B t=52,6^{\circ}$
Inclination angle $B i$ of moist soil:
$\tan B i=V f /\left(V I+V w \cdot p w g / p t g_{90}\right)$
$\tan B i=0,567 /(0,433+0,065 \cdot 1,0 / 3,0)=1,247 \rightarrow$ angle $B i=51,3^{\circ}$
Shear angle si of moist soil:
$\tan s i=(\tan B i) / 2=1,245 / 2=0,624 \rightarrow$ angle $s i=31,9^{\circ}$

In the embankment, the shear plane of moist soil ( $\mathrm{E}-\mathrm{L}$ ) is established. Hereby, and using calculation depth $a=1,00 \mathrm{~m}$, the volume $V e=A e \cdot a$ on the shear plane becomes load (E-C-L). Simultaneously, the absorbed water changes the properties of moist soil, so that the shear angle is changed from si to se.


Fig. 60 shows the embankment plane (C-L), the shear plane of moist soil ( $\mathrm{C}^{\prime}-\mathrm{Ms}$ ), and the shear plane of wet soil ( $D-M^{\prime}$ ).

## Properties of wet soil

Wet density $p n g=p t g+\left(V I_{t} \cdot p w g / V p_{90}\right)=$
Wet density $p n g=1,700+(0,433 \cdot 1,00 / 1,00)=2,133 \mathrm{t} / \mathrm{m}^{3}$
Inclination angle $B n=V f /\left(V I+V I \cdot p w g / p t g_{90}\right)$
$\tan B n=0,567 /(0,433+0,433 \cdot 1,0 / 3,0)=0,982 \rightarrow$ angle $8 n=44,5^{\circ}$
Shear angle sn of the wet soil
$\tan s n=(\tan 8 n) / 2=0,982 / 2=0,491 \rightarrow$ angle $s n=26,2^{\circ}$

## Calculation of the shear angle under load

As shown in Fig. 60, load height $h x / 4=5,00 / 4=1,25 \mathrm{~m}$ must be applied above point $D$, which creates point $C^{\prime}$. The inclination plane of the wet soil under angle $B e=44,5^{\circ}$ must be applied at point $D$, and taken to the reference axis at point $G$.

This enables height $h s=b u \cdot \tan B e=7,50 \cdot 0,982=7,37 \mathrm{~m}$ to be calculated. Inclination plane of the 'wet soil under load' with height $h s^{*}=h s+h x / 4=7,37+1,25=8,62$ m , and inclination angle $b e$ are created from point $C^{\prime}$ up to point $G$.
Angles $B e$ and se can be calculated from tan $B e=h s^{*} / b u=8,62 / 7,50=1,149$ (angle $\left.B e_{3}=49,0^{\circ}\right)$ and $\tan s e=\tan B e / 2=1,149 / 2=0,575\left(\right.$ angle $\left.s e=29,9^{\circ}\right)$.

## Calculation of soil weight sliding from the embankment

It can be assumed that the soil does not loosen up where it slides down. Consequently, the embankment plane ('shear plane under load') with angle se=29,9${ }^{\circ}$ intersects the vertical reference axis $A-B$ at point $M$. The removal/filling area $A l=A r$ can then be calculated from width $b I I=b r r$ and height $h / 2=5,00 / 2=2,50 \mathrm{~m}$ :

Width $b l l=b r r=h / 2(\tan \operatorname{se})-b r=(2,5 / 0,575)-3,75=0,60 \mathrm{~m}$ Area $A l=A r=b r r \cdot h / 4=0,60 \cdot 5,00 / 4=0,75 \mathrm{~m}^{2}$


Fig. 61 shows the embankment plane (C-L), shear plane of moist soil (E-L), and shear plane of wet soil ( $\mathrm{C}^{\prime}-\mathrm{H}^{\prime}$ ).

## Result

The assumed water absorbance of the embankment soil causes an embankment slide, resulting in the 'shear plane under load' with shear angle se $=29,9^{\circ}$. Moreover, per $1,00 \mathrm{~m}$ embankment length, wet soil from area $A l$ will slide into area $A r=0,75$ $\mathrm{m}^{2}$, thereby forming the wet soil's shear plane. This, and other calculations in the 'Earth Pressure Study' and in the book, show that embankment slides \& landslides can be calculated in advance.

### 3.12 Landslide due to excavation at the slope toe

This section is based on the photo in Fig. 62 and an expert opinion from the Dresden University that describes the depicted soil movement in the embankment as the result of 'inadequate soil compaction'. The following calculation shows that the expert seems to have relied on 'gut feeling' rather than taking the real facts into account.

Because the actual soil properties of the embankment are unknown to me, my calculation is based on my long experience in earthworks and road construction as well as the facts revealed by the photo.


Fig. 62 shows the embankment slide due to faults in the slope toe.
The following assumptions are made for the calculation:
Embankment height $h=h x=5,00 \mathrm{~m}$, slope width $b u=7,50 \mathrm{~m}$, and slope inclination 1 : 1,5.
Height $h f=1,40 \mathrm{~m}$, the widths $b f f=2,00 \mathrm{~m}, b f=1,50 \mathrm{~m}$, and $b s=0,50 \mathrm{~m}$ of the fault area $A f$ were measured in an enlargement of the photo.
The same properties are assigned to the soil, as determined in Sect. 3.11 for a moist soil: $8 i=51,3^{\circ}(\tan B i=1,247)$ and $s i=31,9^{\circ}(\tan s i=0,624)$.
First of all, to investigate the soil behaviour due to the fault at the slope toe, the earth block (C-A-B-D), the soil type above height $h x=5,00 \mathrm{~m}$, and the angles $B i$ and si are created. From this earth block, the soil is allowed to slide down along the shear plane of moist soil. This procedure is shown as force wedge ( $D-H-G$ ), whereby wedge area (C-A-M) is seen as the load on the shear plane (D-H) (see Fig. 63). If the load with $h x$ / 4 is applied above point $D$, the 'inclined plane under load' ( $C-G$ ) and the 'shear plane under load' (C-H') become apparent.

Height $h s=b u \cdot \tan B i=7,50 \cdot 1,247=9,35 \mathrm{~m}$ and height $h s^{*}=h s+h x / 4=9,35+$ $1,25=10,60 \mathrm{~m}$ can be calculated by means of embankment width $b u=7,50 \mathrm{~m}$ and inclination angle $B i$. The 'angles under load' $B e$ and se can be determined:

Angle $B e \rightarrow \tan B e=h s^{*} / b u=10,60 / 7,50=1,413 \rightarrow B e=54,7^{\circ}$
Angle se $\rightarrow \tan s e=\tan B e / 2=1,413 / 2=0,707 \rightarrow s e=35,2^{\circ}$
To determine the void volume at the slope toe after the landslide, angle $8 c$ of the embankment slope L'-F as well as shear angle $s x$ must be calculated (see Fig. 63):

Angle $B c \rightarrow \tan B c=h f /(b f f-b f)=1,40 /(2,00-1,50)=2,800 \rightarrow$
Angle $B c=70,3^{\circ}$
Shear angle $s x \rightarrow \tan s x=h / b u=5,00 / 7,50=0,667 \rightarrow$ angle $s x=33,7^{\circ}$.



Fig. 64

Fig. 63 shows the values for calculating the angles.
Fig. 64 shows the values for calculating the dispersal and load weights.

## Calculation of soil movement in the slope

To fill the void at the slope toe and to stabilize the embankment, the soil in the slope has slipped along the determined shear angle $s e=35,2^{\circ}$ (tan $s e=0,707$ ).
To be calculated in the following are width $b b$ of the fracture in the slope, and the areas of load and dispersal $A I$ and $A r$.
Width $b b=h y / 2(\tan s e-\tan s x)=h y / 2(0,707-0,667)=12,5$ hy
Area $A l=12,5 h^{2} / 2=6,25 h^{2}$
Area $A r=h c^{2} /(2 \tan s e)-h c^{2} /(2 \tan B c)$
Area $A r=h c^{2} /(2 \cdot 0,707)-h c^{2} /(2 \cdot 2,800)$
Area $A r=0,707 h c^{2}-0,179 h c^{2}=0,528 h c^{2}$
Height $h c=h s-h y=1,33-h y$
Height hy via area $A r=A r$
$6,25 h y^{2}=0,528 h c^{2}=0,528 \cdot(1,33-h y)^{2}$
$2,5 h y=0,727 \cdot(1,33-h y)^{2}$
2,5 hy $-0,967+0,727$ hy $=0$
$h y=0,967 / 3,23=0,30 \mathrm{~m}$

Height $h c=h s-h y=1,33-0,30=1,03 \mathrm{~m}$
Width $c c=h c / \tan s e-h c / \tan B c$
Width $c c=1,03 / 0,707-1,03 / 2,800=1,09 \mathrm{~m}$
Area $A l=h y \cdot b b / 2=h y \cdot 12,5$ hy $/ 2=$
Area $A I=0,30^{2} \cdot 12,5 / 2=0,56 \mathrm{~m}^{2}$
Area $A r=h c \cdot c c / 2=1,03 \cdot 1,09 / 2=0,56 \mathrm{~m}^{2}$
Width $b b$ via:
hy $/ 2(\tan s e-\tan s x)=12,5$ hy
$b b=12,5 h y=12,5 \cdot 0,30=3,75 \mathrm{~m}$

Width $b s=b b+b f f=3,75+2,00=5,75 \mathrm{~m}$
Width bue $=h / \tan s i=5,00 / 0,624=8,01 \mathrm{~m}$
Width $b l=b u e-b u=8,01-7,50=0,51 \mathrm{~m}$

## Result

By means of the calculated fracture width $b s=5,75 \mathrm{~m}$, which coincides with that in the photo (see Fig. 62), it can be proved that the slope has slipped due to void at the slope toe. Consequently, the expert opinion of the Dresden University, that 'inadequate soil compaction' was the cause of soil slippage in the embankment, must be questioned.

### 3.13 'Earth pressure tutorial' of the TUM - Part 1: Soil properties

The Center for Geotechnics at the Technical University Munich (TUM) published the 'Übung Erddruck' (Earth pressure tutorial) as Paper L. It shows an example of stress behaviour of soils under water. The tutorial's purpose and the properties of the correspondingly selected soil types (sand and clay) are described in Bild L-4. Shown on page 4 of the Paper are the earth stresses $\sigma^{\prime}{ }_{z z}$ and $\sigma^{\prime}{ }_{x x}$ as well as the total stresses for the soils at rest, which are partially under water. To reconstruct the tutorial, the task of determining earth pressure and the graphical representation of the horizontal stress in Bild L-4 will be copied and summarized.

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## Beispiel:

Für den in Bild L-4 dargestellen Baugrund soll die Verteilung des Erdruhedrucks und der gesamten horizontalen Spannungen bis auf Kote - $7,0 \mathrm{~m}$ ermittelt werden.

Zur Ermittlung des Erdruhedrucks muss zunăchst die Verteilung der effektiven Vertikalspannungen berechnet werden. Unter Verwendung des Erdruhedruckbeiwerts lassen sich daraus die effektiven Horizontalspannungen bestimmen.

Zur Vollstăndigkeit wird im Folgenden auch der Verlauf der Porenwasserdrücke und der totalen Spannungen dargestellt. Im Zusammenhang mit der Ermittlung des Erddrucks ist es sinnvoll, effektive Spannungen und Porenwasserdrücke getrennt $z u$ behandeln.


Bild L-4: Baugrundaufbau fûr alle Beispiele zum Erddruck


Bild L-4: Substratum and stress determination of the Center for Geotechnics at the TUM

The 'Earth pressure tutorial' is used to illustrate the differences between stress calculation according to current teachings and force determination according to the new teachings. The first thing that comes apparent, is that task to be carried out in the tutorial does not state for which application the stresses are to be determined. This information would be helpful, because with supporting walls the adjacent water contributes to load dispersal, whilst in the case of piles, the moist density's water gives way under pressure and is therefore not available for load dispersal.

In the substratum diagram, soils whose properties are determined considerably by the water are shown above and below the groundwater level. But there is no information about the water absorbed by the respective soil type. Therefore, to find a common denominator for the soil properties to start with, the densities and inclination angles of sand and clay are calculated according to the new teachings.

## Properties of the sand

Specified for the sand are density $\nu=18 \mathrm{kN} / \mathrm{m}^{3}$ and angle $\varphi^{\prime}=32,5^{\circ}$. By means of density and angle, it is possible to determine the sand's state: dry, moist or wet.
a) Dry sand

Density $p t g=v / \mathrm{g}=18 / 9,807=1,835 \mathrm{t} / \mathrm{m}^{3} \rightarrow\left(y=18,0 \mathrm{kN} / \mathrm{m}^{3}\right)$
Solids volume $V f=V p \cdot p t g / p t g 90=1,00 \cdot 1,835 / 3,00=0,612 \mathrm{~m}^{3}$
Pore volume $V I=V p-V f=1,000-0,612=0,388 \mathrm{~m}^{3}$
Inclination angle $B t=\rightarrow \tan B t=V f / V I=0,612 / 0,388=1,577 \rightarrow B=57,6^{\circ} \rightarrow$
Angle $\alpha=\varphi=90,0^{\circ}-57,6^{\circ}=32,4^{\circ}$

The calculation reveals that for stress determination, the current teachings use dry density $\gamma=18 \mathrm{kN} / \mathrm{m}^{3}$ above and below the groundwater level, although water changes the angle $\alpha=32,5^{\circ}\left(\varphi^{\prime}, \varphi\right)$. There is no explanation, why the soil mobilization usually required by the current teachings, and indicated by angle $\varphi^{\prime}$, is considered to be superfluous for sand.
b) Moist sand

Observations show that soils near the adjacent groundwater must be classified as moist or wet. For the comparison calculation, moist sand is assumed above the groundwater level, whose pores are filled $60 \%$ with water.

Moist density pig $=(V f \cdot 3,0+0,6 \cdot V I \cdot p w g) / V p=0,612 \cdot 3,0+0,6 \cdot 0,388 \cdot 1,0=$ pig $=2,068 \mathrm{t} / \mathrm{m}^{3}$
Angle $B i \rightarrow \tan B i=V f /\left(V I+V l i \cdot p w g / p t g_{90}\right)=0,612 /(0,388+0,233 \cdot 1,0 / 3,0)=$ 1,314 , therefore inclination angle $B i=52,7^{\circ}$.
c) Wet sand

Wet sand would appear above the groundwater level, if water can penetrate into all pores of the sand:
Volume of a soil cube $V p=1,00 \mathrm{~m}^{3}$
Wet density png $=p t g+V I \cdot p w g / V p=1,835+0,388 \cdot 1,0 / 1,0=2,223 \mathrm{t} / \mathrm{m}^{3}$
Angle $B n \rightarrow \tan B n=V f /(V I+V I \cdot p w g / p t g 90)=0,612 /(0,388+0,388 \cdot 1,0 / 3,0)$ $=1,183$, therefore inclination angle $B n=49,8^{\circ}$.
d) Wet sand under water

With wet soil under water, the pore water pressure with volume VIn $=V I \cdot p w g /$ $\operatorname{ptg}_{90}=\mathrm{VI} / 3$ is superposed by the contrarily-acting hydrostatic water pressure, whereby volume Vnw $=V I / 2$ is assigned to the latter. Due to the hydrostatic uplift, the solids volume $V f$ is divided into volume $V f a$ with the water's density, and volume $V f w$ with the rocky ground's density.

## Consequently:

Volume $V f a=1 \cdot V f \cdot p w g / p t g_{98}=1 \cdot 0,612 \cdot 1,0 / 3,00=0,204 \mathrm{~m}^{3}$
Volume $V f w=2 \cdot V f \cdot p w g / p t g_{98}=2 \cdot 0,612 \cdot 1,0 / 3,00=0,408 \mathrm{~m}^{3}$
Inclination angle $8 n w \rightarrow \tan B n w=V f w /(V I+V I / 3-V I / 2)=0,408 / 0,323=$ 1,263 , therefore inclination angle $8 n w=51,6^{\circ}$.
Density pnwg $=[0,408 \cdot 3,0+(0,388-0,388 / 6) \cdot 1,0] / 1,0=1,547 \mathrm{t} / \mathrm{m}^{3}$, or $\mathrm{y}=15,17 \mathrm{kN} / \mathrm{m}^{3}$.

## Properties of the clay

For the clay, the teachings specify wet density $\gamma=20,0 \mathrm{kN} / \mathrm{m}^{3}$ and dry density $\gamma=$ $10,0 \mathrm{kN} / \mathrm{m}^{3}$, and indicated an angle $\varphi^{\prime}=17,5^{\circ}$. If one applies the density specifications, the wet clay must have absorbed 10 kN of water. Therefore, the solids and pore volumes each account for $0,5 \mathrm{~m}^{3}$ of the soil cube with volume $V p=1,00 \mathrm{~m}^{3}$.
a) Wet clay

Wet density $p n g=y / \mathrm{g}=20,0 / 9,807=2,040 \mathrm{t} / \mathrm{m}^{3}$
Solids volume $V f$ can be calculated from the wet density:
$p n g=\left(p t g_{90} \cdot V f+V I \cdot p w g\right) / V p=(3,0 \cdot V f+V I \cdot 1,00) / 1,00=2,040 \mathrm{t} / \mathrm{m}^{3}$, or wet density $\mathrm{y}=20,0 \mathrm{kN} / \mathrm{m}^{3}$

Solids volume $V f=(p n g-V I) / 3=(2,040-0,500) / 3=0,513 \mathrm{~m}^{3}$
Pore volume $V I=V p-V f=1,000-0,513=0,487 \mathrm{~m}^{3}$
Angle $8 n \rightarrow \tan B n=V f /(V I+V I \cdot p w g / p t g 90)=0,513 /(0,487+0,487 \cdot 1,0 / 3,0)$ $=0,790$, therefore inclination angle $B n=38,3^{\circ}$.
b) Wet clay under water

Volume Vfa $=1 \cdot V f \cdot p w g / p t g_{98}=1 \cdot 0,513 \cdot 1,0 / 3,00=0,171 \mathrm{~m}^{3}$
Volume $V f w=2 \cdot V f \cdot p w g / p t g_{98}=2 \cdot 0,513 \cdot 1,0 / 3,00=0,342 \mathrm{~m}^{3}$
Inclination angle Bnw $\rightarrow \tan B n w=V f w /(V I+V I / 3-V I / 2)=$
Inclination angle Bnw $\rightarrow \tan B n w=0,342 /(0,487+0,487 / 6)=0,602 \rightarrow$
Inclination angle $B n w=31,0^{\circ}$.
Wet density $p n w g=[0,342 \cdot 3,0+(0,487-0,487 / 6) \cdot 1,0] / 1,0=1,432 \mathrm{t} / \mathrm{m}^{3}$, or $\mathrm{y}=14,04 \mathrm{kN} / \mathrm{m}^{3}$.

## Result of the soil properties

For force determination according to the new teachings, the following densities and angles were calculated:
a) Above the water level

Sand density $p i g=2,068 \mathrm{t} / \mathrm{m}^{3}\left(\mathrm{y}=20,28 \mathrm{kN} / \mathrm{m}^{3}\right)$ and angle $B i=52,7^{\circ}(\tan B i=1,314)$.
b) For 'wet sand under water':

Density $p n w g=1,547 \mathrm{t} / \mathrm{m}^{3}\left(\mathrm{y}=15,17 \mathrm{kN} / \mathrm{m}^{3}\right)$, angle $B n w=51,6^{\circ}(\tan B n w=1,263)$.
c) For 'wet clay under water':

Density $p n w g=1,432 \mathrm{t} / \mathrm{m}^{3}\left(\mathrm{y}=14,04 \mathrm{kN} / \mathrm{m}^{3}\right)$, angle $B n w=31,0^{\circ}(\tan B n w=0,602)$.

The volumes for wet sand and clay under water are shown in Figs. 65 and 66.


Fig. 65 shows the 'soil band' of the wet sand under water
Fig. 66 shows the 'soil band' of the clay under water

### 3.14 'Earth pressure tutorial' of the TUM - Part 2: Earth pressure (elevation 0,00 up to $-7,00 \mathrm{~m}$ )

The results of stress determination by the Center for Geotechnics at the TUM are shown in Bild L-4. However, the task here is to determine the forces against a perpendicular wall by means of the new teachings, and to compare the results with the stresses determined by the TUM. Hereby, the real soil densities (solids plus water) and inclination angle are used for force determination. First of all, a standing earth wedge with height $h=7,00 \mathrm{~m}$ must be constructed. The wedge widths are calculated via the partial heights and the inclination angle of the adjacent soil. Finally, a fictive earth wedge is formed via the sum of all partial areas, by means of which the force can be determined.

Widths of the earth wedge:
Elevation $-7,00 \mathrm{~m}$ up to $-5,00 \mathrm{~m} \rightarrow 8 n w=31,0^{\circ} \rightarrow \tan B n w=0,602$
$b o_{1}=h_{1} / \tan B n w=2,0 / 0,602=3,32 \mathrm{~m}$
Elevation $-5,00 \mathrm{~m}$ up to $-3,00 \mathrm{~m} \rightarrow B n w=51,6^{\circ} \rightarrow \tan B n w^{\prime}=1,263$
$b o_{2}=b o_{1}+h_{2} / \tan B n w=3,32+2,0 / 1,263=3,32+1,58=4,90 \mathrm{~m}$
Elevation $-3,00 \mathrm{~m}$ up to $-0,00 \rightarrow B i=52,7^{\circ} \rightarrow \tan B i=1,314$
$b b=b o_{3}=b o_{2}+h_{3} / \tan B i=4,90+3,0 / 1,314=4,90+2,28=7,18 \mathrm{~m}$

Areas of the earth wedge:
$A o_{1}=h_{1} \cdot b o_{1} / 2=2,0 \cdot 3,32 / 2=3,32 \mathrm{~m}^{2}$
$A o_{2}=h_{2} \cdot\left(b o_{1}+b o_{2}\right) / 2=2,0 \cdot(3,32+4,90) / 2=8,22 \mathrm{~m}^{2}$
$A o_{3}=h_{3} \cdot\left(b o_{2}+b o_{3}\right) / 2=3,0 \cdot(4,90+7,18) / 2=18,12 \mathrm{~m}^{2}$
$\sum A o=A o_{1}+A o_{2}+A o_{3}=3,32+8,22+18,12=29,66 \mathrm{~m}^{2}$

Fictive earth wedge:
Wedge width $b b=2 \cdot \sum A o / h=2 \cdot 29,66 / 7,0=8,47 \mathrm{~m}$
Angle $8 m=\rightarrow \tan B m=\sum h / b b=7,00 / 8,47=0,826 \rightarrow 6 m=39,6^{\circ}$
Determination of weight forces with calculation depth $a=1,00 \mathrm{~m}$
$G_{1}=A o_{1} \cdot p t w{ }^{*} \cdot \mathrm{~g}=3,32 \cdot 1,432 \cdot 9,807=46,6 \mathrm{kN}$
$G_{2}=A o_{2} \cdot p t w g ' \cdot g=8,22 \cdot 1,547 \cdot 9,807=124,7 \mathrm{kN}$
$G_{3}=A o_{3} \cdot p i g \cdot g=18,12 \cdot 2,068 \cdot 9,807=367,5 \mathrm{kN}$
Weight force $G G=G_{1}+G_{2}+G_{3}=46,6+124,7+367,5=538,8 \mathrm{kN}$
Soil density $p m g=G G / \sum A o=538,8 / 29,66=18,166 \mathrm{kN} / \mathrm{m}^{3} \rightarrow 1,852 \mathrm{t} / \mathrm{m}^{3}$

Earth pressure calculation
Using weight force $G G=538,8 \mathrm{kN}$ and inclination angle $8 m=39,6^{\circ}$
Force index $g m=G G / h=538,8 / 7,00=76,97 \mathrm{kN} / \mathrm{m}$
Normal force $F N=G G \cdot \cos 39,6^{\circ}=538,8 \cdot 0,771=415,4 \mathrm{kN}$
(force meter $f n=F N / g m=415,4 / 76,97=5,40 \mathrm{~m}$ )
Downhill force $F H=538,8 \cdot \sin 39,6^{\circ}=343,4 \mathrm{kN}$
(force meter fh $=F H / g m=343,4 / 76,97=4,46 \mathrm{~m}$ )
Earth pressure force $H f=538,8 \cdot \sin 39,6^{\circ} \cdot \cos 39,6^{\circ}=264,6 \mathrm{kN}$

Earth pressure force $H f=264,6 \mathrm{kN}$ acts against the fictive wall at height $h v=2,84 \mathrm{~m}$.


Fig. 67 shows the stresses of the soil under water according to the current teachings, whereby the number values are claimed to correspond to the total stresses.
Fig. 68 shows the force meter within the earth wedge according to the new teachings with $h=7,00 \mathrm{~m}$, whereby earth pressure force $H f=264,6 \mathrm{kN}$ acts against the wall at height $h v=$ $2,84 \mathrm{~m}$ above the basal plane.

## Comparison of moments

Moment $M b^{\prime}$ of the current teaching is calculated via the total stresses according to Bild L-4, whereby it must be noted that demonstrably no horizontal stresses occur in the basal planes (elevations $-3,-5$ and -7 ):

Moment $\mathrm{Mb}^{\prime}=3,0 \cdot 25,0 \cdot(7,0-2,0) / 2+(25,0+54,3) \cdot 3,0+(71,8+105,8) \cdot 1,0$
Moment $\mathrm{Mb}^{\prime}=187,5+237,9+177,6=603,0 \mathrm{kNm}$

Moment already determined according to new teachings
Moment $\mathrm{Mb}=\mathrm{Hf} \cdot h v=264,6 \cdot 2,84=751,5 \mathrm{kNm}$

## Result

In summary, it can be said that there is a difference of $M B^{*}=751,5-603,0=148,5$ $\mathbf{k N m}$ between moments $M b=751,5 \mathrm{kNm}$ and $M b^{\prime}=603,0 \mathrm{kN}$, leading to the wall being underdimensioned by about 24,0 \%.

Should the 'Supporting wall' project be implemented according to the TUM's specifications, a damage event would occur due to the wall's obvious underdimensioning. And once again, geologists and planners would state: "for stress determination, we observed the specifications in the teachings". The construction
company would refer to their expertise and that they applied the relevant standards. Subsequently, the assessors and chief assessors would carry out disputes. And finally, the media would report about the incompetence of construction workers and botched-up building projects.

## 4 Summary

The 'New Earth Pressure Teachings' are based on the findings that soils in free nature form active and inactive soil bodies, and that the properties of all soils can be calculated. For force/stress determination, two different calculation systems are available, which are arranged along the position of the center of gravity in the soil body. It was found that 'standing earth wedges' with the center of gravity at $2 / 3$ of the height must be seen as active, and 'lying earth wedges' with the center of gravity in the lower third as inactive (see Sect. 2.1 'Operation of an hourglass').

Moreover, the properties of soils can be calculated precisely, regardless of whether they are in the dry, moist or wet state, or are under water. Consequently, the previous use of empiric soil characteristics is superfluous.

Except for a small addition to load dispersal, Coulomb's 'Classical earth pressure teachings', which use the 'physical plane' for force determination, remain valid. With the introduction of 'earth blocks', that can be combined like building bricks for a calculation task, every construction project can be computed, and soil overloads due to the building structures are excluded.

With the "Mohr-Coulomb fracture condition" and "Mohr's stress circle", the current teachings show that they want to use the 'physical plane' for force determination in their calculation method. Therefore, they first determine the weight force from the soil's own weight via the 'standing earth wedge', but then conclude that in addition to the weight force, "the size and direction of shear deformation in the soil must be taken into account", and therefore insert force $Q$ into the inclination/friction plane. [1: P; S. P.2, Pict. P01.40].


Bild 101.70: Mohr'scher Spannungskreis


Pict. P05.50 Section and force polygon .
Moreover, the current teachings are oriented along the water's stress diagram, and therefore rotate the active 'standing' earth wedge into the inactive 'lying' wedge (Pict. P05.50) in order to prove horizontal stress in the basal plane (Pol-Z). As demonstrated with the hourglass, rotation of the soil body through $180^{\circ}$ causes a transition into a different calculation system. By means of auxiliary constructs such as wall friction forces, unequal vectors ( $\delta_{x}$ and $\delta_{z}$ ), creating horizontal stresses, and earth pressure factors, the current teachings attempt to prove the correctness of their stress determination. But none of these measures are supported by the accepted rules of physics


If Pict. 101.70 of Mohr's stress circle is rotated back again, the force relationships of a 'standing' earth wedge are created again.

Consequently, weight force $G$ adopts plane (Z-X), normal force $F N$ adopts plane (Z-Pol), downhill force FH adopts plane (Pol) -X), and earth pressure force $H f$ acts horizontally at height (Pol).

## Conclusion

Current earth pressure teachings abandon the calculation specifications according to Coulomb's 'Classical earth pressure teachings' and the 'physical plane', and adapt the soil's stress behaviour to the behaviour of water. Hereby, the teachings fail to see that soils are decomposition products of rocky ground, and their properties in the dry state are derived from the interaction of solid particles and pore formation.

Furthermore, stress determinations according to current earth pressure teachings reveal that the two contrary calculation methods - which must be applied for 'standing' and 'lying' earth wedges - are unknown to them (see Para. 2.1, Fig. 1, and Sect. 2.4, Figs. 9 and 10). As both calculation methods follow the pure basics of physics, and the current earth pressure teachings have abandoned the physical basis for their stress determination, in my opinion it is reasonable to speak of 'misinterpretations' in the current teachings.

The consequences of these misinterpretations are shown in Pict. 101.70 for 'Mohr's stress circle'. Originally, weight force $G$ was determined from a 'standing' earth wedge (Pict. P05.50), and subsequently the stresses/forces rotated into a 'lying' earth wedge. If the stresses/forces ( $\mathrm{X}-\mathrm{Z}-\mathrm{Pol}$ ) of Mohr's stress circle are rotated back into their initial position (Z-X-Pol), it can be seen that no stress/force corresponds to the calculated stresses in the 'standing' earth wedge; neither in size nor direction.

If damage to structures subjected to soil pressures, such as supporting walls, underground pipes, and subsidence of structures is to be prevented in future, the specifications for stress calculation used in the current teaching must be given up, and Coulomb's 'Classical earth pressure teachings' applied once again

There is justified hope that this concise version of the New Earth Pressure Teachings can trigger a new start in the calculation of earth pressures.


Fig. 11 shows the ,Semicircle of soil types' with the dependencies on inclination angle, earth pressure force, and their thrust heights against a wall.

For explanations, see Sect. 2.5

